함수가 증가 또는 감소가 아니다.
(A function isn't increasing or decresing.)

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Theorem

f isn't increasing or decreasing on $[a, b]$.

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$\Leftrightarrow$

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$\Leftrightarrow \exists\ x_1, x_2, x_3$

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Github:
https://min7014.github.io/math20240228001.html

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