연쇄법칙
(The Chain Rule)

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Theorem

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$$\left[\ g \text{ is differentiable at } x \right.$$

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$$\left[\begin{array}{l} g \textit{ is differentiable at } x \\ f \textit{ is differentiable at } g(x) \\ \\ \\ \end{array}\right.$$

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Home Start End

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Home Start End

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$$F'(x)$$

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$$k \cdot \varepsilon_2(k) = \{f(u+k)-f(u)\} - f'(u)\cdot k \ , \ f(u+k)-f(u) = f'(u)\cdot k + k\cdot\varepsilon_2(k)$$

$$f(u+k)-f(u) = f'(u)\cdot k + k\cdot\varepsilon_2(k)$$

(Let $k = g(x+h)-g(x)$, $g(x+h) = g(x)+k = u+k$)

$$\begin{aligned} f(g(x+h)) - f(g(x)) &= \{f'(u) + \varepsilon_2(g(x+h)-g(x))\}\{g(x+h)-g(x)\} \\ &= \{f'(u) + \varepsilon_2(g(x+h)-g(x))\}\{g'(x)+\varepsilon_1(h)\}\cdot h \end{aligned}$$

$$F'(x) = \lim_{h\to 0} \frac{f(g(x+h))-f(g(x))}{h}$$

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Proof.

$$\varepsilon_1(h) = \begin{cases} \dfrac{g(x+h)-g(x)}{h} - g'(x) & , \quad h \neq 0 \\ 0 & , \quad h = 0 \end{cases} \qquad \varepsilon_1 \text{ is continuous at } h = 0 \ (\because g \text{ is differentiable at } x)$$

$$h \cdot \varepsilon_1(h) = \{g(x+h) - g(x)\} - g'(x) \cdot h \ , \ g(x+h) - g(x) = g'(x) \cdot h + h \cdot \varepsilon_1(h)$$

$$\varepsilon_2(k) = \begin{cases} \dfrac{f(u+k)-f(u)}{k} - f'(u) & , \quad k \neq 0 \\ 0 & , \quad k = 0 \end{cases} \qquad \varepsilon_2 \text{ is continuous at } k = 0 \ (\because f \text{ is differentiable at } u)$$

$$k \cdot \varepsilon_2(k) = \{f(u+k) - f(u)\} - f'(u) \cdot k \ , \ f(u+k) - f(u) = f'(u) \cdot k + k \cdot \varepsilon_2(k)$$

$$\begin{aligned} f(u+k) - f(u) &= f'(u) \cdot k + k \cdot \varepsilon_2(k) \\ &\quad (\text{Let } k = g(x+h) - g(x) \ , \ g(x+h) = g(x) + k = u + k) \end{aligned}$$

$$\begin{aligned} f(g(x+h)) - f(g(x)) &= \{f'(u) + \varepsilon_2(g(x+h) - g(x))\}\{g(x+h) - g(x)\} \\ &= \{f'(u) + \varepsilon_2(g(x+h) - g(x))\}\{g'(x) + \varepsilon_1(h)\} \cdot h \end{aligned}$$

$$\begin{aligned} F'(x) &= \lim_{h \to 0} \frac{f(g(x+h)) - f(g(x))}{h} \\ &= \lim_{h \to 0} \{f'(u) + \varepsilon_2(g(x+h) - g(x))\}\{g'(x) + \varepsilon_1(h)\} \end{aligned}$$

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Proof.

$$\varepsilon_1(h) = \begin{cases} \dfrac{g(x+h)-g(x)}{h} - g'(x) & , \quad h \neq 0 \\ 0 & , \quad h = 0 \end{cases} \qquad \varepsilon_1 \text{ is continuous at } h = 0 \ (\because g \text{ is differentiable at } x)$$

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$$\begin{aligned} f(u+k) - f(u) &= f'(u) \cdot k + k \cdot \varepsilon_2(k) \\ & \quad (\text{Let } k = g(x+h) - g(x) \ , \ g(x+h) = g(x) + k = u + k) \end{aligned}$$

$$\begin{aligned} f(g(x+h)) - f(g(x)) &= \{f'(u) + \varepsilon_2(g(x+h) - g(x))\}\{g(x+h) - g(x)\} \\ &= \{f'(u) + \varepsilon_2(g(x+h) - g(x))\}\{g'(x) + \varepsilon_1(h)\} \cdot h \end{aligned}$$

$$\begin{aligned} F'(x) &= \lim_{h \to 0} \frac{f(g(x+h)) - f(g(x))}{h} \\ &= \lim_{h \to 0} \{f'(u) + \varepsilon_2(g(x+h) - g(x))\}\{g'(x) + \varepsilon_1(h)\} \\ &= f'(u)g'(x) \end{aligned}$$

Proof.

$$\varepsilon_1(h) = \begin{cases} \dfrac{g(x+h)-g(x)}{h} - g'(x) & , \quad h \neq 0 \\ 0 & , \quad h = 0 \end{cases} \quad \varepsilon_1 \text{ is continuous at } h=0 \ (\because g \text{ is differentiable at } x)$$

$$h \cdot \varepsilon_1(h) = \{g(x+h)-g(x)\} - g'(x)\cdot h \ , \ g(x+h)-g(x) = g'(x)\cdot h + h\cdot\varepsilon_1(h)$$

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$$k \cdot \varepsilon_2(k) = \{f(u+k)-f(u)\} - f'(u)\cdot k \ , \ f(u+k)-f(u) = f'(u)\cdot k + k\cdot\varepsilon_2(k)$$

$$\begin{aligned} f(u+k)-f(u) &= f'(u)\cdot k + k\cdot\varepsilon_2(k) \\ & \quad (\text{Let } k = g(x+h)-g(x) \ , \ g(x+h) = g(x)+k = u+k) \\ f(g(x+h))-f(g(x)) &= \{f'(u)+\varepsilon_2(g(x+h)-g(x))\}\{g(x+h)-g(x)\} \\ &= \{f'(u)+\varepsilon_2(g(x+h)-g(x))\}\{g'(x)+\varepsilon_1(h)\}\cdot h \\ F'(x) &= \lim_{h\to 0}\frac{f(g(x+h))-f(g(x))}{h} \\ &= \lim_{h\to 0}\{f'(u)+\varepsilon_2(g(x+h)-g(x))\}\{g'(x)+\varepsilon_1(h)\} \\ &= f'(u)g'(x) = f'(g(x))g'(x) \end{aligned}$$

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Proof.

$$\varepsilon_1(h) = \begin{cases} \dfrac{g(x+h)-g(x)}{h} - g'(x) & , \quad h \neq 0 \\ 0 & , \quad h = 0 \end{cases}$$

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$$\begin{aligned} f(g(x+h)) - f(g(x)) &= \{f'(u) + \varepsilon_2(g(x+h) - g(x))\}\{g(x+h) - g(x)\} \\ &= \{f'(u) + \varepsilon_2(g(x+h) - g(x))\}\{g'(x) + \varepsilon_1(h)\} \cdot h \end{aligned}$$

$$\begin{aligned} F'(x) &= \lim_{h \to 0} \frac{f(g(x+h)) - f(g(x))}{h} \\ &= \lim_{h \to 0} \{f'(u) + \varepsilon_2(g(x+h) - g(x))\}\{g'(x) + \varepsilon_1(h)\} \\ &= f'(u)g'(x) = f'(g(x))g'(x) \end{aligned}$$

$$\therefore F'(x) = f'(g(x))g'(x)$$

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Proof.

$$\varepsilon_1(h) = \begin{cases} \dfrac{g(x+h)-g(x)}{h} - g'(x) & , \quad h \neq 0 \\ 0 & , \quad h = 0 \end{cases} \quad \varepsilon_1 \text{ is continuous at } h = 0 \ (\because g \text{ is differentiable at } x)$$

$$h \cdot \varepsilon_1(h) = \{g(x+h)-g(x)\} - g'(x) \cdot h \ , \ g(x+h)-g(x) = g'(x) \cdot h + h \cdot \varepsilon_1(h)$$

$$\varepsilon_2(k) = \begin{cases} \dfrac{f(u+k)-f(u)}{k} - f'(u) & , \quad k \neq 0 \\ 0 & , \quad k = 0 \end{cases} \quad \varepsilon_2 \text{ is continuous at } k = 0 \ (\because f \text{ is differentiable at } u)$$

$$k \cdot \varepsilon_2(k) = \{f(u+k)-f(u)\} - f'(u) \cdot k \ , \ f(u+k)-f(u) = f'(u) \cdot k + k \cdot \varepsilon_2(k)$$

$$\begin{aligned} f(u+k)-f(u) &= f'(u) \cdot k + k \cdot \varepsilon_2(k) \\ & \quad (\text{Let } k = g(x+h)-g(x) \ , \ g(x+h) = g(x)+k = u+k) \\ f(g(x+h)) - f(g(x)) &= \{f'(u) + \varepsilon_2(g(x+h)-g(x))\}\{g(x+h)-g(x)\} \\ &= \{f'(u) + \varepsilon_2(g(x+h)-g(x))\}\{g'(x) + \varepsilon_1(h)\} \cdot h \\ F'(x) &= \lim_{h \to 0} \frac{f(g(x+h)) - f(g(x))}{h} \\ &= \lim_{h \to 0} \{f'(u) + \varepsilon_2(g(x+h)-g(x))\}\{g'(x) + \varepsilon_1(h)\} \\ &= f'(u)g'(x) = f'(g(x))g'(x) \end{aligned}$$

$$\therefore F'(x) = f'(g(x))g'(x)$$

□

Github:
https://min7014.github.io/math20240219001.html

Click or paste URL into the URL search bar, and you can see a picture moving.