$$\exists f'(a) \Rightarrow \lim_{x \to a} f(x) = f(a)$$

$\exists f'(a) \Rightarrow \lim_{x \to a} f(x) = f(a)$

▸ Start ▸ End

Start End

Theorem

Start End

Theorem

$$\exists f'(a)$$

Start End

Theorem

$$\exists f'(a) \Rightarrow$$

Start End

Theorem

$$\exists f'(a) \Rightarrow \lim_{x \to a} f(x) = f(a)$$

▸ Start ▸ End

Theorem

$$\exists f'(a) \Rightarrow \lim_{x \to a} f(x) = f(a)$$

Proof.

▸ Start ▸ End

Theorem

$$\exists f'(a) \Rightarrow \lim_{x \to a} f(x) = f(a)$$

Proof.

$(\Rightarrow)$

▸ Start ▸ End

Theorem

$$\exists f'(a) \Rightarrow \lim_{x \to a} f(x) = f(a)$$

Proof.

$(\Rightarrow)$

$$\lim_{x \to a} f(x)$$

Start End

Theorem

$$\exists f'(a) \Rightarrow \lim_{x \to a} f(x) = f(a)$$

Proof.

$(\Rightarrow)$

$$\lim_{x \to a} f(x) = \lim_{x \to a} \{f(x)$$

Start End

Theorem

$$\exists f'(a) \Rightarrow \lim_{x \to a} f(x) = f(a)$$

Proof.

$(\Rightarrow)$

$$\lim_{x \to a} f(x) = \lim_{x \to a} \{f(x) - f(a) + f(a)\}$$

Start End

Theorem

$$\exists f'(a) \Rightarrow \lim_{x \to a} f(x) = f(a)$$

Proof.

$(\Rightarrow)$

$$\begin{aligned}\lim_{x \to a} f(x) &= \lim_{x \to a} \{f(x) - f(a) + f(a)\} \\ &= \lim_{x \to a} \left\{ \frac{f(x) - f(a)}{x - a} \right.\end{aligned}$$

Start End

Theorem

$$\exists f'(a) \Rightarrow \lim_{x \to a} f(x) = f(a)$$

Proof.

$(\Rightarrow)$

$$\begin{aligned} \lim_{x \to a} f(x) &= \lim_{x \to a} \{f(x) - f(a) + f(a)\} \\ &= \lim_{x \to a} \left\{ \frac{f(x) - f(a)}{x - a} \times (x - a) \right. \end{aligned}$$

Start End

Theorem

$$\exists f'(a) \Rightarrow \lim_{x \to a} f(x) = f(a)$$

Proof.

$(\Rightarrow)$

$$\begin{aligned} \lim_{x \to a} f(x) &= \lim_{x \to a} \{f(x) - f(a) + f(a)\} \\ &= \lim_{x \to a} \left\{ \frac{f(x) - f(a)}{x - a} \times (x - a) + f(a) \right\} \end{aligned}$$

Start End

Theorem

$$\exists f'(a) \Rightarrow \lim_{x\to a} f(x) = f(a)$$

Proof.

$(\Rightarrow)$

$$\begin{aligned}
\lim_{x\to a} f(x) &= \lim_{x\to a} \{f(x) - f(a) + f(a)\} \\
&= \lim_{x\to a} \left\{ \frac{f(x)-f(a)}{x-a} \times (x-a) + f(a) \right\} \\
&= \lim_{x\to a} \frac{f(x)-f(a)}{x-a}
\end{aligned}$$

Start End

Theorem

$$\exists f'(a) \Rightarrow \lim_{x \to a} f(x) = f(a)$$

Proof.

$(\Rightarrow)$

$$\begin{aligned}
\lim_{x \to a} f(x) &= \lim_{x \to a} \{f(x) - f(a) + f(a)\} \\
&= \lim_{x \to a} \left\{ \frac{f(x) - f(a)}{x - a} \times (x - a) + f(a) \right\} \\
&= \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \times \lim_{x \to a} (x - a)
\end{aligned}$$

Start End

Theorem

$$\exists f'(a) \Rightarrow \lim_{x \to a} f(x) = f(a)$$

Proof.

$(\Rightarrow)$

$$\begin{aligned}
\lim_{x \to a} f(x) &= \lim_{x \to a} \{f(x) - f(a) + f(a)\} \\
&= \lim_{x \to a} \left\{ \frac{f(x) - f(a)}{x - a} \times (x - a) + f(a) \right\} \\
&= \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \times \lim_{x \to a} (x - a) + f(a)
\end{aligned}$$

Start End

Theorem

$$\exists f'(a) \Rightarrow \lim_{x \to a} f(x) = f(a)$$

Proof.

$(\Rightarrow)$

$$\begin{aligned}
\lim_{x \to a} f(x) &= \lim_{x \to a} \{f(x) - f(a) + f(a)\} \\
&= \lim_{x \to a} \left\{ \frac{f(x) - f(a)}{x - a} \times (x - a) + f(a) \right\} \\
&= \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \times \lim_{x \to a} (x - a) + f(a) = f(a)
\end{aligned}$$

Start End

Theorem

$$\exists f'(a) \Rightarrow \lim_{x\to a} f(x) = f(a)$$

Proof.

$(\Rightarrow)$

$$\begin{aligned}
\lim_{x\to a} f(x) &= \lim_{x\to a} \{f(x) - f(a) + f(a)\} \\
&= \lim_{x\to a} \left\{\frac{f(x)-f(a)}{x-a} \times (x-a) + f(a)\right\} \\
&= \lim_{x\to a} \frac{f(x)-f(a)}{x-a} \times \lim_{x\to a}(x-a) + f(a) = f(a)
\end{aligned}$$

$(\not\Leftarrow)$

Start End

Theorem

$$\exists f'(a) \Rightarrow \lim_{x \to a} f(x) = f(a)$$

Proof.

$(\Rightarrow)$

$$\begin{aligned}
\lim_{x \to a} f(x) &= \lim_{x \to a} \{f(x) - f(a) + f(a)\} \\
&= \lim_{x \to a} \left\{ \frac{f(x) - f(a)}{x - a} \times (x - a) + f(a) \right\} \\
&= \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \times \lim_{x \to a} (x - a) + f(a) = f(a)
\end{aligned}$$

$(\not\Leftarrow)$

$$f(x) = |x|$$

Start End

Theorem

$$\exists f'(a) \Rightarrow \lim_{x \to a} f(x) = f(a)$$

Proof.

$(\Rightarrow)$

$$\begin{aligned}
\lim_{x \to a} f(x) &= \lim_{x \to a} \{f(x) - f(a) + f(a)\} \\
&= \lim_{x \to a} \left\{ \frac{f(x) - f(a)}{x - a} \times (x - a) + f(a) \right\} \\
&= \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \times \lim_{x \to a} (x - a) + f(a) = f(a)
\end{aligned}$$

$(\not\Leftarrow)$

$$f(x) = |x| \text{ at } x = 0$$

Start End

Theorem

$$\exists f'(a) \Rightarrow \lim_{x \to a} f(x) = f(a)$$

Proof.

($\Rightarrow$)

$$\begin{aligned}
\lim_{x \to a} f(x) &= \lim_{x \to a} \{f(x) - f(a) + f(a)\} \\
&= \lim_{x \to a} \left\{ \frac{f(x) - f(a)}{x - a} \times (x - a) + f(a) \right\} \\
&= \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \times \lim_{x \to a} (x - a) + f(a) = f(a)
\end{aligned}$$

($\not\Leftarrow$)

$$f(x) = |x| \text{ at } x = 0$$

□

Github:
https://min7014.github.io/math20240131001.html

Click or paste URL into the URL search bar, and you can see a picture moving.