

평균 변화율과 순간 변화율

(The average rate of change and the instantaneous rate of change)

The average rate of change and the instantaneous rate of change

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The average rate of change and the instantaneous rate of change

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Definition

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Definition

$$y = f(x)$$

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▶ End

Definition

$y = f(x)$ is a function.

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▶ End

Definition

$y = f(x)$ is a function.

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

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▶ End

Definition

$y = f(x)$ is a function.

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{\text{The increment of } y}{\text{The increment of } x}$$

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The average rate

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The average rate of change of y

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The average rate of change of y with respect to x

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The average rate of change of y with respect to x over the interval $[x_1, x_2]$ or $[x_2, x_1]$

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$$f'(x_1) =$$

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$$f'(x_1) = \lim$$

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$$f'(x_1) = \lim_{\Delta x \rightarrow 0} \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}$$

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The instantaneous rate

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The instantaneous rate of change of y

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The instantaneous rate of change of y with respect to x at $x = x_1$

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The instantaneous rate of change of y with respect to x at $x = x_1$

The derivative $f'(a)$

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The instantaneous rate of change of y with respect to x at $x = x_1$

The derivative $f'(a)$ is the instantaneous rate

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The instantaneous rate of change of y with respect to x at $x = x_1$

The derivative $f'(a)$ is the instantaneous rate of change of $y = f(x)$

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The instantaneous rate of change of y with respect to x at $x = x_1$

The derivative $f'(a)$ is the instantaneous rate of change of $y = f(x)$ with respect to x when $x = a$.

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Github:

<https://min7014.github.io/math20240129001.html>

Click or paste URL into the URL search bar,
and you can see a picture moving.