

좌극한, 우극한  
(Definiton of One-Sided Limits)

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- $\lim$

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- $\lim_x$

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- $\lim_{x \rightarrow}$

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- $\lim_{x \rightarrow a}$

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- $\lim_{x \rightarrow a} f(x)$



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- $\lim_{x \rightarrow a} f(x) = L$

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- $\lim_{x \rightarrow a} f(x) = L$   
 $\forall$

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- $\lim_{x \rightarrow a} f(x) = L$   
 $\forall \epsilon$

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- $\lim_{x \rightarrow a} f(x) = L$   
 $\forall \epsilon > 0$

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- $\lim_{x \rightarrow a} f(x) = L$   
 $\forall \epsilon > 0,$

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- $\lim_{x \rightarrow a} f(x) = L$   
 $\forall \epsilon > 0, \exists$

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- $\lim_{x \rightarrow a} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta$

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- $\lim_{x \rightarrow a} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$



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- $\lim_{x \rightarrow a} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.

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- $\lim_{x \rightarrow a} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $0 < |x - a| < \delta$

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- $\lim_{x \rightarrow a} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $0 < |x - a| < \delta \Rightarrow$

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- $\lim_{x \rightarrow a} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$

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- $\lim_{x \rightarrow}$

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 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$
- $\lim_{x \rightarrow a^-}$



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- $\lim_{x \rightarrow a} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$
- $\lim_{x \rightarrow a^-} f(x)$

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- $\lim_{x \rightarrow a} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$
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- $\lim_{x \rightarrow a^-} f(x) = L$   
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- $\lim_{x \rightarrow a^-} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $a - \delta < x < a$

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 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $a - \delta < x < a \Rightarrow |f(x) - L| < \epsilon$
- $\lim_{x \rightarrow a^+}$

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- $\lim_{x \rightarrow a} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$
- $\lim_{x \rightarrow a^-} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $a - \delta < x < a \Rightarrow |f(x) - L| < \epsilon$
- $\lim_{x \rightarrow a^+} f(x)$

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- $\lim_{x \rightarrow a} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$
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 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$
- $\lim_{x \rightarrow a^-} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $a - \delta < x < a \Rightarrow |f(x) - L| < \epsilon$
- $\lim_{x \rightarrow a^+} f(x) = L$   
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 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$
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 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $a - \delta < x < a \Rightarrow |f(x) - L| < \epsilon$
- $\lim_{x \rightarrow a^+} f(x) = L$   
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 $\forall \epsilon > 0,$



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 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$
- $\lim_{x \rightarrow a^-} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $a - \delta < x < a \Rightarrow |f(x) - L| < \epsilon$
- $\lim_{x \rightarrow a^+} f(x) = L$   
 $\forall \epsilon > 0, \exists$

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 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$
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 $\forall \epsilon > 0, \exists \delta$

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 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $a - \delta < x < a \Rightarrow |f(x) - L| < \epsilon$
- $\lim_{x \rightarrow a^+} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $a < x < a + \delta$

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- $\lim_{x \rightarrow a} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$
- $\lim_{x \rightarrow a^-} f(x) = L$   
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 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $a < x < a + \delta \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim$



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 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $a < x < a + \delta \Rightarrow |f(x) - L| < \epsilon$
- $\lim_{x \rightarrow a} f(x)$

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 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$
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 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $a < x < a + \delta \Rightarrow |f(x) - L| < \epsilon$
- $\lim_{x \rightarrow a} f(x) = L \Leftrightarrow$

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 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$
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 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$
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 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $a < x < a + \delta \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a} f(x) = L \Leftrightarrow \lim$

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 $\forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } 0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a^-} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } a - \delta < x < a \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a^+} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } a < x < a + \delta \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a} f(x) = L \Leftrightarrow \lim_{x \rightarrow a^-} f(x) = L \text{ and } \lim_{x \rightarrow a^+} f(x) = L$



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- $\bullet \lim_{x \rightarrow a} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } 0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$
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 $\forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } a - \delta < x < a \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a^+} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } a < x < a + \delta \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a} f(x) = L \Leftrightarrow \lim_{x \rightarrow a^-} f(x) = L \text{ and } \lim_{x \rightarrow a^+} f(x) = L$

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 $\forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } 0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$
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 $\forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } a - \delta < x < a \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a^+} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } a < x < a + \delta \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a} f(x) = L \Leftrightarrow \lim_{x \rightarrow a^-} f(x)$

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- $\bullet \lim_{x \rightarrow a} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a^-} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $a - \delta < x < a \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a^+} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $a < x < a + \delta \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a} f(x) = L \Leftrightarrow \lim_{x \rightarrow a^-} f(x) = L$

▶ Start

▶ End

- $\bullet \lim_{x \rightarrow a} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a^-} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $a - \delta < x < a \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a^+} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $a < x < a + \delta \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a} f(x) = L \Leftrightarrow \lim_{x \rightarrow a^-} f(x) = L$  and

▶ Start

▶ End

- $\bullet \lim_{x \rightarrow a} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } 0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a^-} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } a - \delta < x < a \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a^+} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } a < x < a + \delta \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a} f(x) = L \Leftrightarrow \lim_{x \rightarrow a^-} f(x) = L \text{ and } \lim_{x \rightarrow a^+} f(x) = L$

▶ Start

▶ End

- $\bullet \lim_{x \rightarrow a} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a^-} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $a - \delta < x < a \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a^+} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $a < x < a + \delta \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a} f(x) = L \Leftrightarrow \lim_{x \rightarrow a^-} f(x) = L$  and  $\lim_x$

▶ Start

▶ End

- $$\lim_{x \rightarrow a} f(x) = L$$

$$\forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } 0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$$
- $$\lim_{x \rightarrow a^-} f(x) = L$$

$$\forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } a - \delta < x < a \Rightarrow |f(x) - L| < \epsilon$$
- $$\lim_{x \rightarrow a^+} f(x) = L$$

$$\forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } a < x < a + \delta \Rightarrow |f(x) - L| < \epsilon$$
- $$\lim_{x \rightarrow a} f(x) = L \Leftrightarrow \lim_{x \rightarrow a^-} f(x) = L \text{ and } \lim_{x \rightarrow a^+} f(x) = L$$

▶ Start

▶ End

- $\bullet \lim_{x \rightarrow a} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a^-} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $a - \delta < x < a \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a^+} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $a < x < a + \delta \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a} f(x) = L \Leftrightarrow \lim_{x \rightarrow a^-} f(x) = L$  and  $\lim_{x \rightarrow a^+} f(x) = L$



▶ Start

▶ End

- $\bullet \lim_{x \rightarrow a} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a^-} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $a - \delta < x < a \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a^+} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $a < x < a + \delta \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a} f(x) = L \Leftrightarrow \lim_{x \rightarrow a^-} f(x) = L$  and  $\lim_{x \rightarrow a^+} f(x) = L$

▶ Start

▶ End

- $\bullet \lim_{x \rightarrow a} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } 0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a^-} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } a - \delta < x < a \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a^+} f(x) = L$   
 $\forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } a < x < a + \delta \Rightarrow |f(x) - L| < \epsilon$
- $\bullet \lim_{x \rightarrow a} f(x) = L \Leftrightarrow \lim_{x \rightarrow a^-} f(x) = L \text{ and } \lim_{x \rightarrow a^+} f(x) = L$

Github:

<https://min7014.github.io/math20240109001.html>

Click or paste URL into the URL search bar,  
and you can see a picture moving.