압착정리
(The Squeeze Theorem)

The Squeeze Theorem

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Theorem

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Theorem

$f(x) \leq g(x) \leq h(x)(0 < |x - a| < \delta_0)$, $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$

$$\lim_{x \to a} g(x) = L$$

Proof.

$\epsilon > 0$

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$f(x) \leq g(x) \leq h(x)(0 < |x-a| < \delta_0)$, $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$

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$\exists \delta_1 > 0$

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$\exists \delta_1 > 0$ s.t. $0 < |x-a| < \delta_1$

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Proof.

$\epsilon > 0$

$\exists \delta_1 > 0$ s.t. $0 < |x - a| < \delta_1 \Rightarrow$

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$\exists \delta_1 > 0$ s.t. $0 < |x-a| < \delta_1 \Rightarrow |f(x) - L| < \epsilon$

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$\exists \delta_1 > 0$ s.t. $0 < |x-a| < \delta_1 \Rightarrow |f(x) - L| < \epsilon \ (\because \lim_{x \to a} f(x) = L)$

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$\exists \delta_1 > 0$ s.t. $0 < |x-a| < \delta_1 \Rightarrow |f(x) - L| < \epsilon \ (\because \lim_{x \to a} f(x) = L)$, $L - \epsilon < f(x) < L + \epsilon$

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$\exists \delta_2 > 0$ s.t. $0 < |x - a| < \delta_2$

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Theorem

$f(x) \leq g(x) \leq h(x)(0 < |x-a| < \delta_0)$, $\lim_{x \longrightarrow a} f(x) = \lim_{x \longrightarrow a} h(x) = L$

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Proof.

$\epsilon > 0$

$\exists \delta_1 > 0$ s.t. $0 < |x-a| < \delta_1 \Rightarrow |f(x) - L| < \epsilon$ $(\because \lim_{x \longrightarrow a} f(x) = L)$, $L - \epsilon < f(x) < L + \epsilon$

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Proof.

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$\exists \delta_1 > 0 \text{ s.t. } 0 < |x-a| < \delta_1 \Rightarrow |f(x) - L| < \epsilon \ (\because \lim_{x \to a} f(x) = L)$, $L - \epsilon < f(x) < L + \epsilon$

$\exists \delta_2 > 0 \text{ s.t. } 0 < |x-a| < \delta_2 \Rightarrow |h(x) - L| < \epsilon \ (\because \lim_{x \to a} h(x) = L)$, $L - \epsilon < g(x) < L + \epsilon$

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Theorem

$f(x) \leq g(x) \leq h(x)(0 < |x-a| < \delta_0)$, $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$

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Proof.

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$\exists \delta_1 > 0$ s.t. $0 < |x-a| < \delta_1 \Rightarrow |f(x)-L| < \epsilon \; (\because \lim_{x\to a} f(x) = L)$, $L-\epsilon < f(x) < L+\epsilon$

$\exists \delta_2 > 0$ s.t. $0 < |x-a| < \delta_2 \Rightarrow |h(x)-L| < \epsilon \; (\because \lim_{x\to a} h(x) = L)$, $L-\epsilon < g(x) < L+\epsilon$

$\delta = \min\{\delta_0, \delta_1, \delta_2\}$

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$f(x) \leq g(x) \leq h(x)$

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Start End

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$$f(x) \leq g(x) \leq h(x) (0 < |x-a| < \delta_0) \ , \lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$$

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$f(x) \leq g(x) \leq h(x) \ , L - \epsilon < f(x) \leq g(x) \leq h(x) < L + \epsilon \ , \ L - \epsilon < g(x) < L + \epsilon$

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$f(x) \le g(x) \le h(x)$, $L - \epsilon < f(x) \le g(x) \le h(x) < L + \epsilon$, $L - \epsilon < g(x) < L + \epsilon$, $|g(x) - L| < \epsilon$

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$$f(x) \le g(x) \le h(x) (0 < |x-a| < \delta_0) \text{ , } \lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$$

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$\exists \delta_1 > 0 \text{ s.t. } 0 < |x-a| < \delta_1 \Rightarrow |f(x) - L| < \epsilon \ (\because \lim_{x \to a} f(x) = L) \text{ , } L - \epsilon < f(x) < L + \epsilon$

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$\therefore \forall \epsilon > 0 \text{ , } \exists \delta > 0$

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$f(x) \le g(x) \le h(x)$, $L - \epsilon < f(x) \le g(x) \le h(x) < L + \epsilon$, $L - \epsilon < g(x) < L + \epsilon$, $|g(x) - L| < \epsilon$

$\therefore \forall \epsilon > 0$, $\exists \delta > 0$ s.t.

Start End

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$\exists \delta_1 > 0 \text{ s.t. } 0 < |x-a| < \delta_1 \Rightarrow |f(x) - L| < \epsilon\ (\because \lim_{x\to a} f(x) = L)\ ,\ L - \epsilon < f(x) < L + \epsilon$

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$\therefore \forall \epsilon > 0\ ,\ \exists \delta > 0 \text{ s.t.} 0 < |x-a| < \delta$

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$\therefore \forall \epsilon > 0$, $\exists \delta > 0 \text{ s.t.} 0 < |x-a| < \delta \Rightarrow$

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$\exists \delta_1 > 0$ s.t. $0 < |x-a| < \delta_1 \Rightarrow |f(x) - L| < \epsilon \; (\because \lim_{x \to a} f(x) = L)$, $L - \epsilon < f(x) < L + \epsilon$

$\exists \delta_2 > 0$ s.t. $0 < |x-a| < \delta_2 \Rightarrow |h(x) - L| < \epsilon \; (\because \lim_{x \to a} h(x) = L)$, $L - \epsilon < g(x) < L + \epsilon$

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$f(x) \leq g(x) \leq h(x)$, $L - \epsilon < f(x) \leq g(x) \leq h(x) < L + \epsilon$, $L - \epsilon < g(x) < L + \epsilon$, $|g(x) - L| < \epsilon$

$\therefore \forall \epsilon > 0$, $\exists \delta > 0$ s.t. $0 < |x-a| < \delta \Rightarrow |g(x) - L| < \epsilon$ □

Github:
https://min7014.github.io/math20240108001.html

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