# 곱의 극한은 극한의 곱이다.

(The limit of a product is the product of the limits.)

# The limit of a product is the product of the limits.

Start End

# The limit of a product is the product of the limits.

Start End

## Theorem

Start End

**Theorem**

$\lim_{x \to a} f(x) = L$

# The limit of a product is the product of the limits.

Start End

**Theorem**

$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$

Start End

**Theorem**

$\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

$$\lim_{x \to a} \{f(x)g(x)\}$$

# The limit of a product is the product of the limits.

Start End

**Theorem**

$\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

$$\lim_{x \to a} \{f(x)g(x)\} = LM$$

Start End

## Theorem

$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$

$$\lim_{x \to a} \{f(x)g(x)\} = LM$$

Proof.

Start End

### Theorem

$\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

$$\lim_{x \to a} \{f(x)g(x)\} = LM$$

### Proof.

$\epsilon > 0$

Start End

## Theorem

$\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

$$\lim_{x \to a} \{f(x)g(x)\} = LM$$

## Proof.

$\epsilon > 0$

$|f(x)g(x) - LM|$

# The limit of a product is the product of the limits.

▸ Start ▸ End

## Theorem

$\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

$$\lim_{x \to a} \{f(x)g(x)\} = LM$$

## Proof.

$\epsilon > 0$

$$|f(x)g(x) - LM| = |f(x)g(x) - f(x)M + f(x)M - LM|$$

Start End

**Theorem**

$\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

$$\lim_{x \to a} \{f(x)g(x)\} = LM$$

**Proof.**

$\epsilon > 0$

$$\begin{aligned} |f(x)g(x) - LM| &= |f(x)g(x) - f(x)M + f(x)M - LM| \\ &\leq |f(x)g(x) - f(x)M| + |f(x)M - LM| \end{aligned}$$

Start End

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$\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

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$\epsilon > 0$

$$\begin{aligned} |f(x)g(x) - LM| &= |f(x)g(x) - f(x)M + f(x)M - LM| \\ &\leq |f(x)g(x) - f(x)M| + |f(x)M - LM| \ (\because \text{The Triangle Inequality}) \end{aligned}$$

# The limit of a product is the product of the limits.

Start End

## Theorem

$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$

$$\lim_{x \to a} \{f(x)g(x)\} = LM$$

## Proof.

$\epsilon > 0$

$$\begin{aligned} |f(x)g(x) - LM| &= |f(x)g(x) - f(x)M + f(x)M - LM| \\ &\leq |f(x)g(x) - f(x)M| + |f(x)M - LM| \ (\because \text{The Triangle Inequality}) \\ &= |f(x)| \cdot |g(x) - M| + |Mf(x) - ML| \end{aligned}$$

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$\lim_{x\to a} f(x) = L$, $\lim_{x\to a} g(x) = M$

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$$\begin{aligned} |f(x)g(x) - LM| &= |f(x)g(x) - f(x)M + f(x)M - LM| \\ &\leq |f(x)g(x) - f(x)M| + |f(x)M - LM| \ (\because \text{The Triangle Inequality}) \\ &= |f(x)| \cdot |g(x) - M| + |Mf(x) - ML| \end{aligned}$$

$\exists\delta_1 > 0$

# The limit of a product is the product of the limits.

Start End

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$$\lim_{x \to a} \{f(x)g(x)\} = LM$$

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$\epsilon > 0$

$$\begin{aligned} |f(x)g(x) - LM| &= |f(x)g(x) - f(x)M + f(x)M - LM| \\ &\leq |f(x)g(x) - f(x)M| + |f(x)M - LM| \ (\because \text{The Triangle Inequality}) \\ &= |f(x)| \cdot |g(x) - M| + |Mf(x) - ML| \end{aligned}$$

$\exists \delta_1 > 0$ s.t.

# The limit of a product is the product of the limits.

Start End

## Theorem

$\lim_{x \longrightarrow a} f(x) = L$, $\lim_{x \longrightarrow a} g(x) = M$

$$\lim_{x \longrightarrow a} \{f(x)g(x)\} = LM$$

## Proof.

$\epsilon > 0$

$$\begin{aligned} |f(x)g(x) - LM| &= |f(x)g(x) - f(x)M + f(x)M - LM| \\ &\leq |f(x)g(x) - f(x)M| + |f(x)M - LM| \ (\because \text{The Triangle Inequality}) \\ &= |f(x)| \cdot |g(x) - M| + |Mf(x) - ML| \end{aligned}$$

$\exists \delta_1 > 0$ s.t. $0 < |x - a| < \delta_1$

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Start End

## Theorem

$\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

$$\lim_{x \to a} \{f(x)g(x)\} = LM$$

## Proof.

$\epsilon > 0$

$$\begin{aligned} |f(x)g(x) - LM| &= |f(x)g(x) - f(x)M + f(x)M - LM| \\ &\leq |f(x)g(x) - f(x)M| + |f(x)M - LM| \ (\because \text{ The Triangle Inequality}) \\ &= |f(x)| \cdot |g(x) - M| + |Mf(x) - ML| \end{aligned}$$

$\exists \delta_1 > 0$ s.t. $0 < |x - a| < \delta_1 \Rightarrow$

# The limit of a product is the product of the limits.

Start End

## Theorem

$\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

$$\lim_{x \to a} \{f(x)g(x)\} = LM$$

## Proof.

$\epsilon > 0$

$$\begin{aligned} |f(x)g(x) - LM| &= |f(x)g(x) - f(x)M + f(x)M - LM| \\ &\le |f(x)g(x) - f(x)M| + |f(x)M - LM| \ (\because \text{The Triangle Inequality}) \\ &= |f(x)| \cdot |g(x) - M| + |Mf(x) - ML| \end{aligned}$$

$\exists \delta_1 > 0$ s.t. $0 < |x - a| < \delta_1 \Rightarrow |f(x) - L| < 1$

Start End

## Theorem

$\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

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## Proof.

$\epsilon > 0$

$$\begin{aligned} |f(x)g(x) - LM| &= |f(x)g(x) - f(x)M + f(x)M - LM| \\ &\leq |f(x)g(x) - f(x)M| + |f(x)M - LM| \ (\because \text{The Triangle Inequality}) \\ &= |f(x)| \cdot |g(x) - M| + |Mf(x) - ML| \end{aligned}$$

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# The limit of a product is the product of the limits.

Start End

## Theorem

$\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

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$\epsilon > 0$

$$\begin{aligned} |f(x)g(x) - LM| &= |f(x)g(x) - f(x)M + f(x)M - LM| \\ &\leq |f(x)g(x) - f(x)M| + |f(x)M - LM| \ (\because \text{The Triangle Inequality}) \\ &= |f(x)| \cdot |g(x) - M| + |Mf(x) - ML| \end{aligned}$$

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Start End

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$\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

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$$\begin{aligned} |f(x)g(x) - LM| &= |f(x)g(x) - f(x)M + f(x)M - LM| \\ &\leq |f(x)g(x) - f(x)M| + |f(x)M - LM| \text{ (}\because\text{ The Triangle Inequality)} \\ &= |f(x)| \cdot |g(x) - M| + |Mf(x) - ML| \end{aligned}$$

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$|f(x)|$

Start End

## Theorem

$\lim_{x \longrightarrow a} f(x) = L, \lim_{x \longrightarrow a} g(x) = M$

$$\lim_{x \longrightarrow a} \{f(x)g(x)\} = LM$$

## Proof.

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$|f(x)| = |f(x) - L + L|$

Start End

## Theorem

$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$

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$|f(x)| = |f(x) - L + L| \leq |f(x) - L| + |L|$

Start End

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$\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

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$|f(x)| = |f(x) - L + L| \leq |f(x) - L| + |L| < 1 + |L|$

Start End

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$|f(x)| = |f(x) - L + L| \leq |f(x) - L| + |L| < 1 + |L|$

$\exists \delta_2 > 0$

# The limit of a product is the product of the limits.

Start End

## Theorem

$\lim_{x\to a} f(x) = L$, $\lim_{x\to a} g(x) = M$

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$\exists \delta_2 > 0$ s.t.

Start End

**Theorem**

$\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

$$\lim_{x \to a} \{f(x)g(x)\} = LM$$

**Proof.**

$\epsilon > 0$

$$\begin{aligned} |f(x)g(x) - LM| &= |f(x)g(x) - f(x)M + f(x)M - LM| \\ &\leq |f(x)g(x) - f(x)M| + |f(x)M - LM| \ (\because \text{The Triangle Inequality}) \\ &= |f(x)| \cdot |g(x) - M| + |Mf(x) - ML| \end{aligned}$$

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$|f(x)| = |f(x) - L + L| \leq |f(x) - L| + |L| < 1 + |L|$

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# The limit of a product is the product of the limits.

▸ Start ▸ End

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$\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

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## Proof.

$\epsilon > 0$

$$\begin{aligned} |f(x)g(x) - LM| &= |f(x)g(x) - f(x)M + f(x)M - LM| \\ &\leq |f(x)g(x) - f(x)M| + |f(x)M - LM| \ (\because \text{The Triangle Inequality}) \\ &= |f(x)| \cdot |g(x) - M| + |Mf(x) - ML| \end{aligned}$$

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Start End

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$\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

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$$\begin{aligned} |f(x)g(x) - LM| &= |f(x)g(x) - f(x)M + f(x)M - LM| \\ &\leq |f(x)g(x) - f(x)M| + |f(x)M - LM| \ (\because \text{The Triangle Inequality}) \\ &= |f(x)| \cdot |g(x) - M| + |Mf(x) - ML| \end{aligned}$$

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$|f(x)| = |f(x) - L + L| \leq |f(x) - L| + |L| < 1 + |L|$

$\exists \delta_2 > 0$ s.t. $0 < |x - a| < \delta_2 \Rightarrow |g(x) - M| < \dfrac{\epsilon}{2(1 + |L|)}$

# The limit of a product is the product of the limits.

Start End

## Theorem

$\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

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$\epsilon > 0$

$$\begin{aligned} |f(x)g(x) - LM| &= |f(x)g(x) - f(x)M + f(x)M - LM| \\ &\leq |f(x)g(x) - f(x)M| + |f(x)M - LM| \; (\because \text{The Triangle Inequality}) \\ &= |f(x)| \cdot |g(x) - M| + |Mf(x) - ML| \end{aligned}$$

$\exists \delta_1 > 0$ s.t. $0 < |x - a| < \delta_1 \Rightarrow |f(x) - L| < 1$ $(\because \lim_{x \to a} f(x) = L)$ $(\because$ The Triangle Inequality$)$

$|f(x)| = |f(x) - L + L| \leq |f(x) - L| + |L| < 1 + |L|$

$\exists \delta_2 > 0$ s.t. $0 < |x - a| < \delta_2 \Rightarrow |g(x) - M| < \dfrac{\epsilon}{2(1 + |L|)}$ $(\because \lim_{x \to a} g(x) = M)$

▸ Start ▸ End

## Theorem

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## Proof.

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$\exists \delta_3 > 0$

# The limit of a product is the product of the limits.

Start End

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Start End

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$\exists \delta_3 > 0$ s.t. $0 < |x - a| < \delta_3$

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Start End

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## Theorem

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$$\begin{aligned} |f(x)g(x) - LM| &= |f(x)g(x) - f(x)M + f(x)M - LM| \\ &\leq |f(x)g(x) - f(x)M| + |f(x)M - LM| \text{ } (\because \text{The Triangle Inequality}) \\ &= |f(x)| \cdot |g(x) - M| + |Mf(x) - ML| \end{aligned}$$

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$\exists \delta_3 > 0$ s.t. $0 < |x - a| < \delta_3 \Rightarrow |Mf(x) - ML| < \dfrac{\epsilon}{2}$

# The limit of a product is the product of the limits.

Start End

## Theorem

$\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

$$\lim_{x \to a} \{f(x)g(x)\} = LM$$

## Proof.

$\epsilon > 0$

$$\begin{aligned} |f(x)g(x) - LM| &= |f(x)g(x) - f(x)M + f(x)M - LM| \\ &\leq |f(x)g(x) - f(x)M| + |f(x)M - LM| \ (\because \text{The Triangle Inequality}) \\ &= |f(x)| \cdot |g(x) - M| + |Mf(x) - ML| \end{aligned}$$

$\exists \delta_1 > 0$ s.t. $0 < |x - a| < \delta_1 \Rightarrow |f(x) - L| < 1$ ($\because \lim_{x \to a} f(x) = L$) ($\because$ The Triangle Inequality)

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Github:
https://min7014.github.io/math20240104001.html

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