

The limit of a sum is the sum of the limits.

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(The limit of a sum is the sum of the limits.)

The limit of a sum is the sum of the limits.

▶ Start

▶ End

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▶ Start

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Theorem

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▶ Start

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Theorem

$$\lim_{x \rightarrow a} f(x) = L$$

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▶ Start

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Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M$$

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$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M$$

$$\lim_{x \rightarrow a} \{f(x) + g(x)\}$$

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▶ Start

▶ End

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M$$

$$\lim_{x \rightarrow a} \{f(x) + g(x)\} = L + M$$

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$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M$$

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Proof.

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Proof.

$$\epsilon > 0$$

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Proof.

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$$|\{f(x) + g(x)\} - (L + M)|$$

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Proof.

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$$|\{f(x) + g(x)\} - (L + M)| = |\{f(x) - L\} + \{g(x) - M\}|$$

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The limit of a sum is the sum of the limits.

▶ Start

▶ End

Theorem

$$\lim_{x \rightarrow a} f(x) = L, \lim_{x \rightarrow a} g(x) = M$$

$$\lim_{x \rightarrow a} \{f(x) + g(x)\} = L + M$$

Proof.

$$\epsilon > 0$$

$$\begin{aligned} |\{f(x) + g(x)\} - (L + M)| &= |\{f(x) - L\} + \{g(x) - M\}| \\ &\leq |f(x) - L| + |g(x) - M| \quad (\because \text{The Triangle Inequality}) \end{aligned}$$

$$\exists \delta_1 > 0 \text{ s.t. } 0 < |x - a| < \delta_1 \Rightarrow |f(x) - L| < \frac{\epsilon}{2} \quad (\because \lim_{x \rightarrow a} f(x) = L)$$

$$\exists \delta_2 > 0 \text{ s.t. } 0 < |x - a| < \delta_2 \Rightarrow |g(x) - M| < \frac{\epsilon}{2} \quad (\because \lim_{x \rightarrow a} g(x) = M)$$

$$\delta = \min(\delta_1, \delta_2)$$

$$0 < |x - a| < \delta \Rightarrow |\{f(x) + g(x)\} - (L + M)| < \epsilon$$

$$\therefore \forall \epsilon > 0, \exists \delta > 0 \text{ s.t. } 0 < |x - a| < \delta \Rightarrow |\{f(x) + g(x)\} - (L + M)| < \epsilon$$

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The limit of a sum is the sum of the limits.

Github:

<https://min7014.github.io/math20231128001.html>

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