$$\sum_{r=0}^{n} r^2 \cdot {}_n\mathrm{C}_r \, a^r b^{n-r} = na(na+b)(a+b)^{n-2}$$

$\sum_{r=0}^{n} r^2 \cdot {}_n\mathrm{C}_r\, a^r b^{n-r} = na(na+b)(a+b)^{n-2}$

Start End

$\sum_{r=0}^{n} r^2 \cdot {}_n\mathrm{C}_r \, a^r b^{n-r} = na(na+b)(a+b)^{n-2}$

Start End

$$\sum_{r=0}^{n} r \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r$$

Start End

$$\sum_{r=0}^{n} r \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r \quad = \quad nat(at+b)^{n-1}$$

▸ Start ▸ End

$$\sum_{r=0}^{n} r \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r \qquad = \qquad nat(at+b)^{n-1}$$

$$\sum_{r=1}^{n}$$

Start End

$$\begin{aligned} \sum_{r=0}^{n} r \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\ \sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^{r-1} & \end{aligned}$$

Start End

$$\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^{r-1} &= na
\end{aligned}$$

Start End

$$\begin{aligned} \sum_{r=0}^{n} r \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\ \sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} \end{aligned}$$

Start End

$$\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} + nat
\end{aligned}$$

$\sum_{r=0}^{n} r^2 \cdot {}_n C_r \, a^r b^{n-r} = na(na+b)(a+b)^{n-2}$

Start End

$$\sum_{r=0}^{n} r \cdot {}_n C_r \cdot a^r b^{n-r} t^r = nat(at+b)^{n-1}$$

$$\sum_{r=1}^{n} r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} = na(at+b)^{n-1} + nat(n-1)(at+b)^{n-2} a$$

Start End

$$\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} + nat(n-1)(at+b)^{n-2}a \\
&= na(at+b)^{n-2}
\end{aligned}$$

$\sum_{r=0}^{n} r^2 \cdot {}_n\mathrm{C}_r\, a^r b^{n-r} = na(na+b)(a+b)^{n-2}$

Start End

$$
\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} + nat(n-1)(at+b)^{n-2}a \\
&= na(at+b)^{n-2}\left\{(at+b)\right.
\end{aligned}
$$

$\sum_{r=0}^{n} r^2 \cdot {}_n\mathrm{C}_r \, a^r b^{n-r} = na(na+b)(a+b)^{n-2}$

Start End

$$
\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} + nat(n-1)(at+b)^{n-2}a \\
&= na(at+b)^{n-2}\{(at+b)+t(n-1)a\}
\end{aligned}
$$

$\sum_{r=0}^{n} r^2 \cdot {}_n\mathrm{C}_r \; a^r b^{n-r} = na(na+b)(a+b)^{n-2}$

Start End

$$\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} + nat(n-1)(at+b)^{n-2}a \\
&= na(at+b)^{n-2}\{(at+b) + t(n-1)a\} \\
&= na(at+b)^{n-2}
\end{aligned}$$

Start End

$$\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} + nat(n-1)(at+b)^{n-2}a \\
&= na(at+b)^{n-2}\{(at+b)+t(n-1)a\} \\
&= na(at+b)^{n-2}(at+b+nat-at)
\end{aligned}$$

$\sum_{r=0}^{n} r^2 \cdot {}_n\mathrm{C}_r \, a^r b^{n-r} = na(na+b)(a+b)^{n-2}$

Start End

$$\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} + nat(n-1)(at+b)^{n-2}a \\
&= na(at+b)^{n-2}\{(at+b) + t(n-1)a\} \\
&= na(at+b)^{n-2}(at+b+nat-at) \\
&= na(at+b)^{n-2}
\end{aligned}$$

$\sum_{r=0}^{n} r^2 \cdot {}_n\mathrm{C}_r \, a^r b^{n-r} = na(na+b)(a+b)^{n-2}$

Start End

$$\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} + nat(n-1)(at+b)^{n-2}a \\
&= na(at+b)^{n-2}\{(at+b) + t(n-1)a\} \\
&= na(at+b)^{n-2}(at+b+nat-at) \\
&= na(at+b)^{n-2}(b+nat)
\end{aligned}$$

$\sum_{r=0}^{n} r^2 \cdot {}_n\mathrm{C}_r \, a^r b^{n-r} = na(na+b)(a+b)^{n-2}$

Start End

$$\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} + nat(n-1)(at+b)^{n-2} a \\
&= na(at+b)^{n-2} \{(at+b) + t(n-1)a\} \\
&= na(at+b)^{n-2}(at+b+nat-at) \\
&= na(at+b)^{n-2}(b+nat) = na(at+b)^{n-2}
\end{aligned}$$

$\sum_{r=0}^{n} r^2 \cdot {}_nC_r \, a^r b^{n-r} = na(na+b)(a+b)^{n-2}$

Start End

$$\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_nC_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_nC_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} + nat(n-1)(at+b)^{n-2}a \\
&= na(at+b)^{n-2}\{(at+b)+t(n-1)a\} \\
&= na(at+b)^{n-2}(at+b+nat-at) \\
&= na(at+b)^{n-2}(b+nat) = na(at+b)^{n-2}(nat+b)
\end{aligned}$$

$\sum_{r=0}^{n} r^2 \cdot {}_n\mathrm{C}_r \, a^r b^{n-r} = na(na+b)(a+b)^{n-2}$

Start End

$$
\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} + nat(n-1)(at+b)^{n-2}a \\
&= na(at+b)^{n-2}\{(at+b)+t(n-1)a\} \\
&= na(at+b)^{n-2}(at+b+nat-at) \\
&= na(at+b)^{n-2}(b+nat) = na(at+b)^{n-2}(nat+b) \\
&= na(nat+b)(at+b)^{n-2}
\end{aligned}
$$

▸ Start ▸ End

$$\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} + nat(n-1)(at+b)^{n-2}a \\
&= na(at+b)^{n-2}\{(at+b)+t(n-1)a\} \\
&= na(at+b)^{n-2}(at+b+nat-at) \\
&= na(at+b)^{n-2}(b+nat) = na(at+b)^{n-2}(nat+b) \\
&= na(nat+b)(at+b)^{n-2}
\end{aligned}$$

$$\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r}$$

▸ Start ▸ End

$$
\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} + nat(n-1)(at+b)^{n-2}a \\
&= na(at+b)^{n-2}\left\{(at+b)+t(n-1)a\right\} \\
&= na(at+b)^{n-2}(at+b+nat-at) \\
&= na(at+b)^{n-2}(b+nat) = na(at+b)^{n-2}(nat+b) \\
&= na(nat+b)(at+b)^{n-2}
\end{aligned}
$$

$$\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r$$

$$\sum_{r=0}^{n} r^2 \cdot {}_n\mathrm{C}_r\, a^r b^{n-r} = na(na+b)(a+b)^{n-2}$$

Start End

$$\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} + nat(n-1)(at+b)^{n-2}a \\
&= na(at+b)^{n-2}\{(at+b)+t(n-1)a\} \\
&= na(at+b)^{n-2}(at+b+nat-at) \\
&= na(at+b)^{n-2}(b+nat) = na(at+b)^{n-2}(nat+b) \\
&= na(nat+b)(at+b)^{n-2} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= na
\end{aligned}$$

Start End

$$
\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} + nat(n-1)(at+b)^{n-2}a \\
&= na(at+b)^{n-2}\{(at+b)+t(n-1)a\} \\
&= na(at+b)^{n-2}(at+b+nat-at) \\
&= na(at+b)^{n-2}(b+nat) = na(at+b)^{n-2}(nat+b) \\
&= na(nat+b)(at+b)^{n-2} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat
\end{aligned}
$$

Start End

$$\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} + nat(n-1)(at+b)^{n-2}a \\
&= na(at+b)^{n-2}\{(at+b)+t(n-1)a\} \\
&= na(at+b)^{n-2}(at+b+nat-at) \\
&= na(at+b)^{n-2}(b+nat) = na(at+b)^{n-2}(nat+b) \\
&= na(nat+b)(at+b)^{n-2} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(nat+b)(at+b)^{n-2}
\end{aligned}$$

Start End

$$
\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} + nat(n-1)(at+b)^{n-2}a \\
&= na(at+b)^{n-2}\left\{(at+b) + t(n-1)a\right\} \\
&= na(at+b)^{n-2}(at+b+nat-at) \\
&= na(at+b)^{n-2}(b+nat) = na(at+b)^{n-2}(nat+b) \\
&= na(nat+b)(at+b)^{n-2} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(nat+b)(at+b)^{n-2} = \sum_{r=0}^{n}
\end{aligned}
$$

$\sum_{r=0}^{n} r^2 \cdot {}_n C_r \; a^r b^{n-r} = na(na+b)(a+b)^{n-2}$

Start End

$$\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_n C_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} + nat(n-1)(at+b)^{n-2}a \\
&= na(at+b)^{n-2}\left\{(at+b) + t(n-1)a\right\} \\
&= na(at+b)^{n-2}(at+b+nat-at) \\
&= na(at+b)^{n-2}(b+nat) = na(at+b)^{n-2}(nat+b) \\
&= na(nat+b)(at+b)^{n-2} \\
\sum_{r=1}^{n} r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r &= nat(nat+b)(at+b)^{n-2} = \sum_{r=0}^{n} r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r
\end{aligned}$$

$\sum_{r=0}^{n} r^2 \cdot {}_n\mathrm{C}_r \, a^r b^{n-r} = na(na+b)(a+b)^{n-2}$

Start End

$$
\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} + nat(n-1)(at+b)^{n-2}a \\
&= na(at+b)^{n-2}\{(at+b) + t(n-1)a\} \\
&= na(at+b)^{n-2}(at+b+nat-at) \\
&= na(at+b)^{n-2}(b+nat) = na(at+b)^{n-2}(nat+b) \\
&= na(nat+b)(at+b)^{n-2} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(nat+b)(at+b)^{n-2} = \sum_{r=0}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r
\end{aligned}
$$

$\therefore$

Start End

$$\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} + nat(n-1)(at+b)^{n-2}a \\
&= na(at+b)^{n-2}\{(at+b)+t(n-1)a\} \\
&= na(at+b)^{n-2}(at+b+nat-at) \\
&= na(at+b)^{n-2}(b+nat) = na(at+b)^{n-2}(nat+b) \\
&= na(nat+b)(at+b)^{n-2} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(nat+b)(at+b)^{n-2} = \sum_{r=0}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r
\end{aligned}$$

$$\therefore \sum_{r=0}^{n}$$

$\sum_{r=0}^{n} r^2 \cdot {}_n C_r \; a^r b^{n-r} = na(na+b)(a+b)^{n-2}$

Start End

$$
\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_n C_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} + nat(n-1)(at+b)^{n-2} a \\
&= na(at+b)^{n-2} \{(at+b) + t(n-1)a\} \\
&= na(at+b)^{n-2}(at+b+nat-at) \\
&= na(at+b)^{n-2}(b+nat) = na(at+b)^{n-2}(nat+b) \\
&= na(nat+b)(at+b)^{n-2} \\
\sum_{r=1}^{n} r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r &= nat(nat+b)(at+b)^{n-2} = \sum_{r=0}^{n} r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r
\end{aligned}
$$

$$\therefore \sum_{r=0}^{n} r^2$$

▸ Start ▸ End

$$
\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} + nat(n-1)(at+b)^{n-2}a \\
&= na(at+b)^{n-2}\{(at+b) + t(n-1)a\} \\
&= na(at+b)^{n-2}(at+b+nat-at) \\
&= na(at+b)^{n-2}(b+nat) = na(at+b)^{n-2}(nat+b) \\
&= na(nat+b)(at+b)^{n-2} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(nat+b)(at+b)^{n-2} = \sum_{r=0}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r
\end{aligned}
$$

$$\therefore \sum_{r=0}^{n} r^2 \cdot {}_n\mathrm{C}_r$$

$\sum_{r=0}^{n} r^2 \cdot {}_n C_r \, a^r b^{n-r} = na(na+b)(a+b)^{n-2}$

Start End

$$
\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_n C_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} + nat(n-1)(at+b)^{n-2} a \\
&= na(at+b)^{n-2}\{(at+b) + t(n-1)a\} \\
&= na(at+b)^{n-2}(at+b+nat-at) \\
&= na(at+b)^{n-2}(b+nat) = na(at+b)^{n-2}(nat+b) \\
&= na(nat+b)(at+b)^{n-2} \\
\sum_{r=1}^{n} r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r &= nat(nat+b)(at+b)^{n-2} = \sum_{r=0}^{n} r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r
\end{aligned}
$$

$$\therefore \sum_{r=0}^{n} r^2 \cdot {}_n C_r \, a^r$$

$\sum_{r=0}^{n} r^2 \cdot {}_n\mathrm{C}_r \, a^r b^{n-r} = na(na+b)(a+b)^{n-2}$

Start End

$$
\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} + nat(n-1)(at+b)^{n-2}a \\
&= na(at+b)^{n-2}\{(at+b) + t(n-1)a\} \\
&= na(at+b)^{n-2}(at+b+nat-at) \\
&= na(at+b)^{n-2}(b+nat) = na(at+b)^{n-2}(nat+b) \\
&= na(nat+b)(at+b)^{n-2} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(nat+b)(at+b)^{n-2} = \sum_{r=0}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r
\end{aligned}
$$

$$\therefore \sum_{r=0}^{n} r^2 \cdot {}_n\mathrm{C}_r \, a^r b^{n-r}$$

$\sum_{r=0}^{n} r^2 \cdot {}_n C_r \, a^r b^{n-r} = na(na+b)(a+b)^{n-2}$

▸ Start ▸ End

$$
\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_n C_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} + nat(n-1)(at+b)^{n-2} a \\
&= na(at+b)^{n-2} \{(at+b) + t(n-1)a\} \\
&= na(at+b)^{n-2}(at+b+nat-at) \\
&= na(at+b)^{n-2}(b+nat) = na(at+b)^{n-2}(nat+b) \\
&= na(nat+b)(at+b)^{n-2} \\
\sum_{r=1}^{n} r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r &= nat(nat+b)(at+b)^{n-2} = \sum_{r=0}^{n} r^2 \cdot {}_n C_r \cdot a^r b^{n-r} t^r
\end{aligned}
$$

$$\therefore \sum_{r=0}^{n} r^2 \cdot {}_n C_r \, a^r b^{n-r} = na$$

▸ Start ▸ End

$$\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} + nat(n-1)(at+b)^{n-2}a \\
&= na(at+b)^{n-2}\{(at+b)+t(n-1)a\} \\
&= na(at+b)^{n-2}(at+b+nat-at) \\
&= na(at+b)^{n-2}(b+nat) = na(at+b)^{n-2}(nat+b) \\
&= na(nat+b)(at+b)^{n-2} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(nat+b)(at+b)^{n-2} = \sum_{r=0}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r
\end{aligned}$$

$$\therefore \sum_{r=0}^{n} r^2 \cdot {}_n\mathrm{C}_r \; a^r b^{n-r} = na(na+b)$$

Start End

$$\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} + nat(n-1)(at+b)^{n-2}a \\
&= na(at+b)^{n-2}\left\{(at+b) + t(n-1)a\right\} \\
&= na(at+b)^{n-2}(at+b+nat-at) \\
&= na(at+b)^{n-2}(b+nat) = na(at+b)^{n-2}(nat+b) \\
&= na(nat+b)(at+b)^{n-2} \\
\sum_{r=1}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r &= nat(nat+b)(at+b)^{n-2} = \sum_{r=0}^{n} r^2 \cdot {}_n\mathrm{C}_r \cdot a^r b^{n-r} t^r
\end{aligned}$$

$$\therefore \sum_{r=0}^{n} r^2 \cdot {}_n\mathrm{C}_r \; a^r b^{n-r} = na(na+b)(a+b)$$

$\sum_{r=0}^{n} r^2 \cdot {}_nC_r \, a^r b^{n-r} = na(na+b)(a+b)^{n-2}$

Start End

$$
\begin{aligned}
\sum_{r=0}^{n} r \cdot {}_nC_r \cdot a^r b^{n-r} t^r &= nat(at+b)^{n-1} \\
\sum_{r=1}^{n} r^2 \cdot {}_nC_r \cdot a^r b^{n-r} t^{r-1} &= na(at+b)^{n-1} + nat(n-1)(at+b)^{n-2}a \\
&= na(at+b)^{n-2}\{(at+b)+t(n-1)a\} \\
&= na(at+b)^{n-2}(at+b+nat-at) \\
&= na(at+b)^{n-2}(b+nat) = na(at+b)^{n-2}(nat+b) \\
&= na(nat+b)(at+b)^{n-2} \\
\sum_{r=1}^{n} r^2 \cdot {}_nC_r \cdot a^r b^{n-r} t^r &= nat(nat+b)(at+b)^{n-2} = \sum_{r=0}^{n} r^2 \cdot {}_nC_r \cdot a^r b^{n-r} t^r
\end{aligned}
$$

$$\therefore \sum_{r=0}^{n} r^2 \cdot {}_nC_r \, a^r b^{n-r} = na(na+b)(a+b)^{n-2}$$

Github:
https://min7014.github.io/math20230619001.html

Click or paste URL into the URL search bar,
and you can see a picture moving.