

극좌표계에서의 중적분 (Double Integrals in Polar Coordinates)

Double Integrals in Polar Coordinates

▶ Start

▶ End

Double Integrals in Polar Coordinates

▶ Start

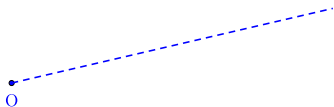
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Double Integrals in Polar Coordinates

▶ Start

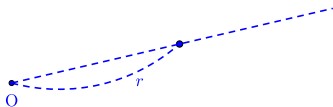
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Double Integrals in Polar Coordinates

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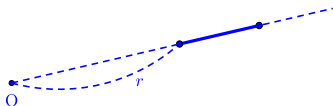
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Double Integrals in Polar Coordinates

▶ Start

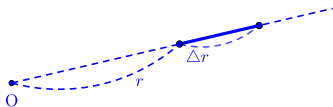
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Double Integrals in Polar Coordinates

▶ Start

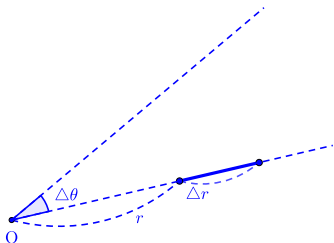
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Double Integrals in Polar Coordinates

► Start

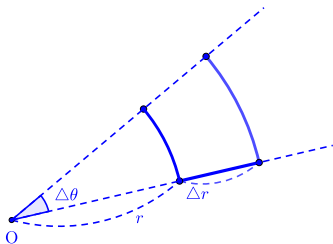
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Double Integrals in Polar Coordinates

▶ Start

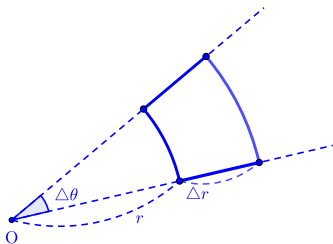
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Double Integrals in Polar Coordinates

▶ Start

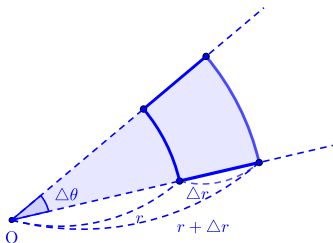
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Double Integrals in Polar Coordinates

▶ Start

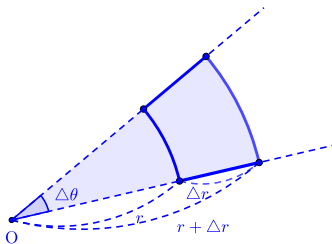
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Double Integrals in Polar Coordinates

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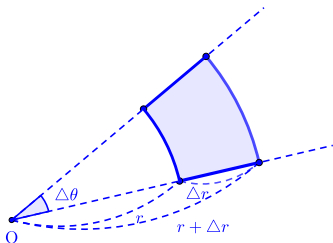


$$\frac{1}{2}(r + \Delta r)^2 \Delta\theta$$

Double Integrals in Polar Coordinates

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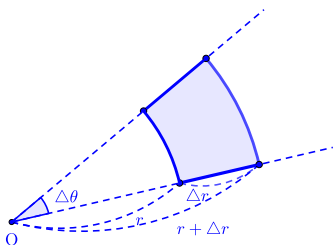


$$\frac{1}{2}(r + \Delta r)^2 \Delta\theta$$

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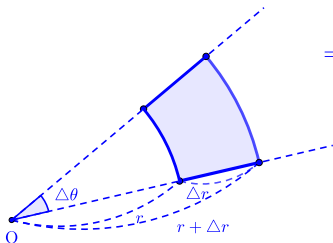


$$\frac{1}{2}(r + \Delta r)^2 \Delta\theta - \frac{1}{2}r^2 \Delta\theta$$

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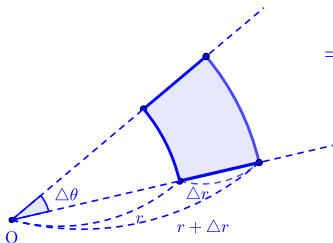


$$\begin{aligned} & \frac{1}{2}(r + \Delta r)^2 \Delta\theta - \frac{1}{2}r^2 \Delta\theta \\ &= \frac{1}{2}(r + \Delta r + r)(r + \Delta r - r)\Delta\theta \end{aligned}$$

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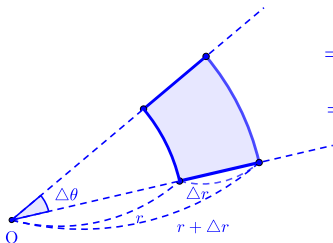


$$\begin{aligned} & \frac{1}{2}(r + \Delta r)^2 \Delta\theta - \frac{1}{2}r^2 \Delta\theta \\ &= \frac{1}{2}(2r + \Delta r)\Delta r \Delta\theta \end{aligned}$$

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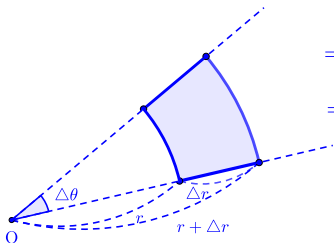


$$\begin{aligned} & \frac{1}{2}(r + \Delta r)^2\Delta\theta - \frac{1}{2}r^2\Delta\theta \\ &= \frac{1}{2}(2r + \Delta r)\Delta r\Delta\theta \\ &= \left(r + \frac{\Delta r}{2}\right)\Delta r\Delta\theta \end{aligned}$$

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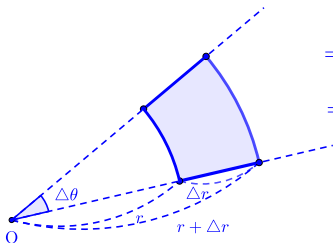


$$\begin{aligned} & \frac{1}{2}(r + \Delta r)^2 \Delta\theta - \frac{1}{2}r^2 \Delta\theta \\ &= \frac{1}{2}(2r + \Delta r) \Delta r \Delta\theta \\ &= \left(r + \frac{\Delta r}{2}\right) \Delta r \Delta\theta \\ & \quad (\Delta r)^2 + (\Delta\theta)^2 \rightarrow 0 \end{aligned}$$

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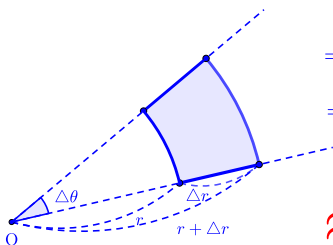


$$\begin{aligned} & \frac{1}{2}(r + \Delta r)^2 \Delta\theta - \frac{1}{2}r^2 \Delta\theta \\ &= \frac{1}{2}(2r + \Delta r) \Delta r \Delta\theta \\ &= \left(r + \frac{\Delta r}{2}\right) \Delta r \Delta\theta \\ & \quad \downarrow (\Delta r)^2 + (\Delta\theta)^2 \rightarrow 0 \end{aligned}$$

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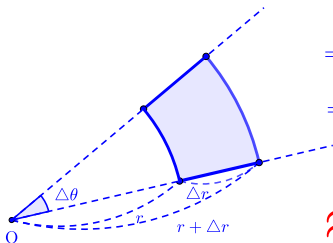


$$\begin{aligned} & \frac{1}{2}(r + \Delta r)^2 \Delta\theta - \frac{1}{2}r^2 \Delta\theta \\ &= \frac{1}{2}(2r + \Delta r)\Delta r \Delta\theta \\ &= \left(r + \frac{\Delta r}{2}\right)\Delta r \Delta\theta \\ & \quad \downarrow (\Delta r)^2 + (\Delta\theta)^2 \rightarrow 0 \\ & \approx r \end{aligned}$$

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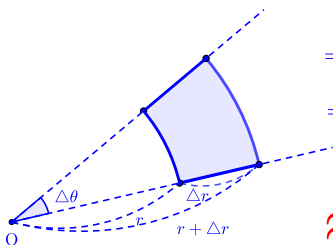


$$\begin{aligned} & \frac{1}{2}(r + \Delta r)^2 \Delta\theta - \frac{1}{2}r^2 \Delta\theta \\ &= \frac{1}{2}(2r + \Delta r) \Delta r \Delta\theta \\ &= \left(r + \frac{\Delta r}{2}\right) \Delta r \Delta\theta \\ & \quad \downarrow (\Delta r)^2 + (\Delta\theta)^2 \rightarrow 0 \\ & \approx r dr \end{aligned}$$

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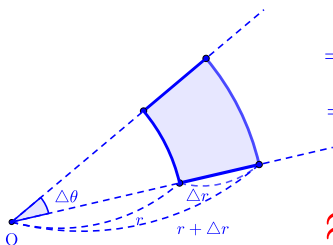


$$\begin{aligned} & \frac{1}{2}(r + \Delta r)^2 \Delta\theta - \frac{1}{2}r^2 \Delta\theta \\ &= \frac{1}{2}(2r + \Delta r) \Delta r \Delta\theta \\ &= \left(r + \frac{\Delta r}{2}\right) \Delta r \Delta\theta \\ & \quad \downarrow (\Delta r)^2 + (\Delta\theta)^2 \rightarrow 0 \\ & \approx r dr d\theta \end{aligned}$$

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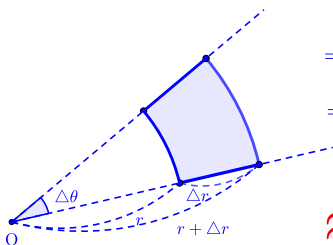
$$\begin{aligned} & \frac{1}{2}(r + \Delta r)^2 \Delta\theta - \frac{1}{2}r^2 \Delta\theta \\ &= \frac{1}{2}(2r + \Delta r)\Delta r \Delta\theta \\ &= \left(r + \frac{\Delta r}{2}\right)\Delta r \Delta\theta \\ & \quad \downarrow (\Delta r)^2 + (\Delta\theta)^2 \rightarrow 0 \\ & \approx r dr d\theta \end{aligned}$$

Let $T(r, \theta) = (r \cos \theta, r \sin \theta)$

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$$\begin{aligned} & \frac{1}{2}(r + \Delta r)^2 \Delta\theta - \frac{1}{2}r^2 \Delta\theta \\ &= \frac{1}{2}(2r + \Delta r)\Delta r \Delta\theta \\ &= \left(r + \frac{\Delta r}{2}\right)\Delta r \Delta\theta \\ & \quad \downarrow (\Delta r)^2 + (\Delta\theta)^2 \rightarrow 0 \\ & \approx r dr d\theta \end{aligned}$$

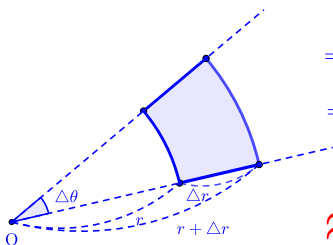
Let $T(r, \theta) = (r \cos \theta, r \sin \theta)$

$$\iint f(x, y) dx dy$$

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$$\begin{aligned} & \frac{1}{2}(r + \Delta r)^2 \Delta\theta - \frac{1}{2}r^2 \Delta\theta \\ &= \frac{1}{2}(2r + \Delta r)\Delta r \Delta\theta \\ &= \left(r + \frac{\Delta r}{2}\right)\Delta r \Delta\theta \\ & \quad \downarrow (\Delta r)^2 + (\Delta\theta)^2 \rightarrow 0 \\ & \approx r dr d\theta \end{aligned}$$

Let $T(r, \theta) = (r \cos \theta, r \sin \theta)$

$$\iint f(x, y) dx dy = \iint f(T(r, \theta)) r dr d\theta$$

Github:

<https://min7014.github.io/math20230615001.html>

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and you can see a picture moving.