

삼각함수의 3배각 공식

(Triple Angle Formula for Trigonometric Functions)

Triple Angle Formula for Trigonometric Functions

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$$\sin 3\alpha =$$

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$$\sin 3\alpha = 3 \sin \alpha - 4 \sin^3 \alpha$$

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$$\sin 3\alpha = 3 \sin \alpha - 4 \sin^3 \alpha$$

▶ proof

$$\cos 3\alpha =$$

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$$\sin 3\alpha = 3 \sin \alpha - 4 \sin^3 \alpha$$

▶ proof

$$\cos 3\alpha = 4 \cos^3 \alpha - 3 \cos \alpha$$

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$$\sin 3\alpha = 3 \sin \alpha - 4 \sin^3 \alpha$$

▶ proof

$$\cos 3\alpha = 4 \cos^3 \alpha - 3 \cos \alpha$$

▶ proof

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$$\sin 3\alpha =$$

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$$\sin 3\alpha = \sin(\alpha + 2\alpha)$$

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$$\begin{aligned}\sin 3\alpha &= \sin(\alpha + 2\alpha) \\ &= \end{aligned}$$

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$$\begin{aligned}\sin 3\alpha &= \sin(\alpha + 2\alpha) \\ &= \sin \alpha \cos 2\alpha + \cos \alpha \sin 2\alpha\end{aligned}$$

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$$\begin{aligned}\sin 3\alpha &= \sin(\alpha + 2\alpha) \\ &= \sin \alpha \cos 2\alpha + \cos \alpha \sin 2\alpha \\ &= \end{aligned}$$

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$$\begin{aligned}\sin 3\alpha &= \sin(\alpha + 2\alpha) \\&= \sin \alpha \cos 2\alpha + \cos \alpha \sin 2\alpha \\&= \sin \alpha(1 - 2 \sin^2 \alpha) + \cos \alpha(2 \sin \alpha \cos \alpha)\end{aligned}$$

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$$\begin{aligned}\sin 3\alpha &= \sin(\alpha + 2\alpha) \\&= \sin \alpha \cos 2\alpha + \cos \alpha \sin 2\alpha \\&= \sin \alpha(1 - 2 \sin^2 \alpha) + \cos \alpha(2 \sin \alpha \cos \alpha) \\&= \end{aligned}$$

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$$\begin{aligned}\sin 3\alpha &= \sin(\alpha + 2\alpha) \\&= \sin \alpha \cos 2\alpha + \cos \alpha \sin 2\alpha \\&= \sin \alpha(1 - 2 \sin^2 \alpha) + \cos \alpha(2 \sin \alpha \cos \alpha) \\&= \sin \alpha(1 - 2 \sin^2 \alpha) + 2 \sin \alpha \cos^2 \alpha\end{aligned}$$

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$$\begin{aligned}\sin 3\alpha &= \sin(\alpha + 2\alpha) \\&= \sin \alpha \cos 2\alpha + \cos \alpha \sin 2\alpha \\&= \sin \alpha(1 - 2 \sin^2 \alpha) + \cos \alpha(2 \sin \alpha \cos \alpha) \\&= \sin \alpha(1 - 2 \sin^2 \alpha) + 2 \sin \alpha \cos^2 \alpha \\&=\end{aligned}$$

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$$\begin{aligned}\sin 3\alpha &= \sin(\alpha + 2\alpha) \\&= \sin \alpha \cos 2\alpha + \cos \alpha \sin 2\alpha \\&= \sin \alpha(1 - 2 \sin^2 \alpha) + \cos \alpha(2 \sin \alpha \cos \alpha) \\&= \sin \alpha(1 - 2 \sin^2 \alpha) + 2 \sin \alpha \cos^2 \alpha \\&= \sin \alpha(1 - 2 \sin^2 \alpha) + 2 \sin \alpha(1 - \sin^2 \alpha)\end{aligned}$$

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$$\begin{aligned}\sin 3\alpha &= \sin(\alpha + 2\alpha) \\&= \sin \alpha \cos 2\alpha + \cos \alpha \sin 2\alpha \\&= \sin \alpha(1 - 2 \sin^2 \alpha) + \cos \alpha(2 \sin \alpha \cos \alpha) \\&= \sin \alpha(1 - 2 \sin^2 \alpha) + 2 \sin \alpha \cos^2 \alpha \\&= \sin \alpha(1 - 2 \sin^2 \alpha) + 2 \sin \alpha(1 - \sin^2 \alpha) \\&= \end{aligned}$$

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$$\begin{aligned}\sin 3\alpha &= \sin(\alpha + 2\alpha) \\&= \sin \alpha \cos 2\alpha + \cos \alpha \sin 2\alpha \\&= \sin \alpha(1 - 2 \sin^2 \alpha) + \cos \alpha(2 \sin \alpha \cos \alpha) \\&= \sin \alpha(1 - 2 \sin^2 \alpha) + 2 \sin \alpha \cos^2 \alpha \\&= \sin \alpha(1 - 2 \sin^2 \alpha) + 2 \sin \alpha(1 - \sin^2 \alpha) \\&= \sin \alpha - 2 \sin^3 \alpha + 2 \sin \alpha - 2 \sin^3 \alpha\end{aligned}$$

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$$\begin{aligned}\sin 3\alpha &= \sin(\alpha + 2\alpha) \\&= \sin \alpha \cos 2\alpha + \cos \alpha \sin 2\alpha \\&= \sin \alpha(1 - 2 \sin^2 \alpha) + \cos \alpha(2 \sin \alpha \cos \alpha) \\&= \sin \alpha(1 - 2 \sin^2 \alpha) + 2 \sin \alpha \cos^2 \alpha \\&= \sin \alpha(1 - 2 \sin^2 \alpha) + 2 \sin \alpha(1 - \sin^2 \alpha) \\&= \sin \alpha - 2 \sin^3 \alpha + 2 \sin \alpha - 2 \sin^3 \alpha \\&= \end{aligned}$$

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$$\begin{aligned}\sin 3\alpha &= \sin(\alpha + 2\alpha) \\&= \sin \alpha \cos 2\alpha + \cos \alpha \sin 2\alpha \\&= \sin \alpha(1 - 2 \sin^2 \alpha) + \cos \alpha(2 \sin \alpha \cos \alpha) \\&= \sin \alpha(1 - 2 \sin^2 \alpha) + 2 \sin \alpha \cos^2 \alpha \\&= \sin \alpha(1 - 2 \sin^2 \alpha) + 2 \sin \alpha(1 - \sin^2 \alpha) \\&= \sin \alpha - 2 \sin^3 \alpha + 2 \sin \alpha - 2 \sin^3 \alpha \\&= 3 \sin \alpha - 4 \sin^3 \alpha\end{aligned}$$

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$$\begin{aligned}\sin 3\alpha &= \sin(\alpha + 2\alpha) \\&= \sin \alpha \cos 2\alpha + \cos \alpha \sin 2\alpha \\&= \sin \alpha(1 - 2 \sin^2 \alpha) + \cos \alpha(2 \sin \alpha \cos \alpha) \\&= \sin \alpha(1 - 2 \sin^2 \alpha) + 2 \sin \alpha \cos^2 \alpha \\&= \sin \alpha(1 - 2 \sin^2 \alpha) + 2 \sin \alpha(1 - \sin^2 \alpha) \\&= \sin \alpha - 2 \sin^3 \alpha + 2 \sin \alpha - 2 \sin^3 \alpha \\&= 3 \sin \alpha - 4 \sin^3 \alpha\end{aligned}$$

$$\therefore \sin 3\alpha =$$

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$$\begin{aligned}\sin 3\alpha &= \sin(\alpha + 2\alpha) \\&= \sin \alpha \cos 2\alpha + \cos \alpha \sin 2\alpha \\&= \sin \alpha(1 - 2 \sin^2 \alpha) + \cos \alpha(2 \sin \alpha \cos \alpha) \\&= \sin \alpha(1 - 2 \sin^2 \alpha) + 2 \sin \alpha \cos^2 \alpha \\&= \sin \alpha(1 - 2 \sin^2 \alpha) + 2 \sin \alpha(1 - \sin^2 \alpha) \\&= \sin \alpha - 2 \sin^3 \alpha + 2 \sin \alpha - 2 \sin^3 \alpha \\&= 3 \sin \alpha - 4 \sin^3 \alpha\end{aligned}$$

$$\therefore \sin 3\alpha = 3 \sin \alpha - 4 \sin^3 \alpha$$

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$$\cos 3\alpha =$$

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$$\cos 3\alpha = \cos(\alpha + 2\alpha)$$

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$$\begin{aligned}\cos 3\alpha &= \cos(\alpha + 2\alpha) \\ &= \end{aligned}$$

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$$\begin{aligned}\cos 3\alpha &= \cos(\alpha + 2\alpha) \\ &= \cos \alpha \cos 2\alpha - \sin \alpha \sin 2\alpha\end{aligned}$$

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$$\begin{aligned}\cos 3\alpha &= \cos(\alpha + 2\alpha) \\ &= \cos \alpha \cos 2\alpha - \sin \alpha \sin 2\alpha \\ &= \end{aligned}$$

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$$\begin{aligned}\cos 3\alpha &= \cos(\alpha + 2\alpha) \\ &= \cos \alpha \cos 2\alpha - \sin \alpha \sin 2\alpha \\ &= \cos \alpha\end{aligned}$$

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$$\begin{aligned}\cos 3\alpha &= \cos(\alpha + 2\alpha) \\&= \cos \alpha \cos 2\alpha - \sin \alpha \sin 2\alpha \\&= \cos \alpha(2 \cos^2 \alpha - 1) - \sin \alpha(2 \sin \alpha \cos \alpha)\end{aligned}$$

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$$\begin{aligned}\cos 3\alpha &= \cos(\alpha + 2\alpha) \\&= \cos \alpha \cos 2\alpha - \sin \alpha \sin 2\alpha \\&= \cos \alpha(2 \cos^2 \alpha - 1) - \sin \alpha(2 \sin \alpha \cos \alpha) \\&= \end{aligned}$$

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$$\begin{aligned}\cos 3\alpha &= \cos(\alpha + 2\alpha) \\&= \cos \alpha \cos 2\alpha - \sin \alpha \sin 2\alpha \\&= \cos \alpha(2 \cos^2 \alpha - 1) - \sin \alpha(2 \sin \alpha \cos \alpha) \\&= \cos \alpha(2 \cos^2 \alpha - 1) - 2 \sin^2 \alpha \cos \alpha\end{aligned}$$

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$$\begin{aligned}\cos 3\alpha &= \cos(\alpha + 2\alpha) \\&= \cos \alpha \cos 2\alpha - \sin \alpha \sin 2\alpha \\&= \cos \alpha(2 \cos^2 \alpha - 1) - \sin \alpha(2 \sin \alpha \cos \alpha) \\&= \cos \alpha(2 \cos^2 \alpha - 1) - 2 \sin^2 \alpha \cos \alpha \\&=\end{aligned}$$

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$$\begin{aligned}\cos 3\alpha &= \cos(\alpha + 2\alpha) \\&= \cos \alpha \cos 2\alpha - \sin \alpha \sin 2\alpha \\&= \cos \alpha(2 \cos^2 \alpha - 1) - \sin \alpha(2 \sin \alpha \cos \alpha) \\&= \cos \alpha(2 \cos^2 \alpha - 1) - 2 \sin^2 \alpha \cos \alpha \\&= \cos \alpha(2 \cos^2 \alpha - 1) - 2(1 - \cos^2 \alpha) \cos \alpha\end{aligned}$$

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$$\begin{aligned}\cos 3\alpha &= \cos(\alpha + 2\alpha) \\&= \cos \alpha \cos 2\alpha - \sin \alpha \sin 2\alpha \\&= \cos \alpha(2 \cos^2 \alpha - 1) - \sin \alpha(2 \sin \alpha \cos \alpha) \\&= \cos \alpha(2 \cos^2 \alpha - 1) - 2 \sin^2 \alpha \cos \alpha \\&= \cos \alpha(2 \cos^2 \alpha - 1) - 2(1 - \cos^2 \alpha) \cos \alpha \\&= \end{aligned}$$

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$$\begin{aligned}\cos 3\alpha &= \cos(\alpha + 2\alpha) \\&= \cos \alpha \cos 2\alpha - \sin \alpha \sin 2\alpha \\&= \cos \alpha(2 \cos^2 \alpha - 1) - \sin \alpha(2 \sin \alpha \cos \alpha) \\&= \cos \alpha(2 \cos^2 \alpha - 1) - 2 \sin^2 \alpha \cos \alpha \\&= \cos \alpha(2 \cos^2 \alpha - 1) - 2(1 - \cos^2 \alpha) \cos \alpha \\&= 2 \cos^3 \alpha - \cos \alpha - 2 \cos \alpha + 2 \cos^3 \alpha\end{aligned}$$

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$$\begin{aligned}\cos 3\alpha &= \cos(\alpha + 2\alpha) \\&= \cos \alpha \cos 2\alpha - \sin \alpha \sin 2\alpha \\&= \cos \alpha(2 \cos^2 \alpha - 1) - \sin \alpha(2 \sin \alpha \cos \alpha) \\&= \cos \alpha(2 \cos^2 \alpha - 1) - 2 \sin^2 \alpha \cos \alpha \\&= \cos \alpha(2 \cos^2 \alpha - 1) - 2(1 - \cos^2 \alpha) \cos \alpha \\&= 2 \cos^3 \alpha - \cos \alpha - 2 \cos \alpha + 2 \cos^3 \alpha \\&= \end{aligned}$$

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$$\begin{aligned}\cos 3\alpha &= \cos(\alpha + 2\alpha) \\&= \cos \alpha \cos 2\alpha - \sin \alpha \sin 2\alpha \\&= \cos \alpha(2 \cos^2 \alpha - 1) - \sin \alpha(2 \sin \alpha \cos \alpha) \\&= \cos \alpha(2 \cos^2 \alpha - 1) - 2 \sin^2 \alpha \cos \alpha \\&= \cos \alpha(2 \cos^2 \alpha - 1) - 2(1 - \cos^2 \alpha) \cos \alpha \\&= 2 \cos^3 \alpha - \cos \alpha - 2 \cos \alpha + 2 \cos^3 \alpha \\&= 4 \cos^3 \alpha - 3 \cos \alpha\end{aligned}$$

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$$\begin{aligned}\cos 3\alpha &= \cos(\alpha + 2\alpha) \\&= \cos \alpha \cos 2\alpha - \sin \alpha \sin 2\alpha \\&= \cos \alpha(2 \cos^2 \alpha - 1) - \sin \alpha(2 \sin \alpha \cos \alpha) \\&= \cos \alpha(2 \cos^2 \alpha - 1) - 2 \sin^2 \alpha \cos \alpha \\&= \cos \alpha(2 \cos^2 \alpha - 1) - 2(1 - \cos^2 \alpha) \cos \alpha \\&= 2 \cos^3 \alpha - \cos \alpha - 2 \cos \alpha + 2 \cos^3 \alpha \\&= 4 \cos^3 \alpha - 3 \cos \alpha\end{aligned}$$

$$\therefore \cos 3\alpha =$$

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$$\begin{aligned}\cos 3\alpha &= \cos(\alpha + 2\alpha) \\&= \cos \alpha \cos 2\alpha - \sin \alpha \sin 2\alpha \\&= \cos \alpha(2 \cos^2 \alpha - 1) - \sin \alpha(2 \sin \alpha \cos \alpha) \\&= \cos \alpha(2 \cos^2 \alpha - 1) - 2 \sin^2 \alpha \cos \alpha \\&= \cos \alpha(2 \cos^2 \alpha - 1) - 2(1 - \cos^2 \alpha) \cos \alpha \\&= 2 \cos^3 \alpha - \cos \alpha - 2 \cos \alpha + 2 \cos^3 \alpha \\&= 4 \cos^3 \alpha - 3 \cos \alpha\end{aligned}$$

$$\therefore \cos 3\alpha = 4 \cos^3 \alpha - 3 \cos \alpha$$

Github:

<https://min7014.github.io/math20230420001.html>

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and you can see a picture moving.