

회전이동을 이용한 삼각함수의 덧셈공식 유도

(Derivation of Addition Formula of Trigonometric Function using Rotational Movement)

Derivation of Addition Formula of Trigonometric Function using Rotational Movement

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Derivation of Addition Formula of Trigonometric Function using Rotational Movement

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$$\begin{pmatrix} \cos(\alpha + \beta) & -\sin(\alpha + \beta) \\ \sin(\alpha + \beta) & \cos(\alpha + \beta) \end{pmatrix} =$$

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$$\begin{pmatrix} \cos(\alpha + \beta) & -\sin(\alpha + \beta) \\ \sin(\alpha + \beta) & \cos(\alpha + \beta) \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}$$

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$$\begin{pmatrix} \cos(\alpha + \beta) & -\sin(\alpha + \beta) \\ \sin(\alpha + \beta) & \cos(\alpha + \beta) \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix}$$

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$$\begin{aligned} \begin{pmatrix} \cos(\alpha + \beta) & -\sin(\alpha + \beta) \\ \sin(\alpha + \beta) & \cos(\alpha + \beta) \end{pmatrix} &= \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \\ &= \begin{pmatrix} \cos \alpha \cos \beta - \sin \alpha \sin \beta & -\sin \alpha \cos \beta - \cos \alpha \sin \beta \\ \sin \alpha \cos \beta + \cos \alpha \sin \beta & \cos \alpha \cos \beta - \sin \alpha \sin \beta \end{pmatrix} \end{aligned}$$

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$$\begin{aligned} \begin{pmatrix} \cos(\alpha + \beta) & -\sin(\alpha + \beta) \\ \sin(\alpha + \beta) & \cos(\alpha + \beta) \end{pmatrix} &= \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \\ &= \begin{pmatrix} \cos \alpha \cos \beta - \sin \alpha \sin \beta & -\sin \alpha \cos \beta - \cos \alpha \sin \beta \\ \sin \alpha \cos \beta + \cos \alpha \sin \beta & \cos \alpha \cos \beta - \sin \alpha \sin \beta \end{pmatrix} \\ \sin(\alpha + \beta) &= \end{aligned}$$

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$$\begin{aligned} \begin{pmatrix} \cos(\alpha + \beta) & -\sin(\alpha + \beta) \\ \sin(\alpha + \beta) & \cos(\alpha + \beta) \end{pmatrix} &= \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \\ &= \begin{pmatrix} \cos \alpha \cos \beta - \sin \alpha \sin \beta & -\sin \alpha \cos \beta - \cos \alpha \sin \beta \\ \sin \alpha \cos \beta + \cos \alpha \sin \beta & \cos \alpha \cos \beta - \sin \alpha \sin \beta \end{pmatrix} \\ \sin(\alpha + \beta) &= \sin \alpha \cos \beta + \cos \alpha \sin \beta \end{aligned}$$

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$$\begin{pmatrix} \cos(\alpha + \beta) & -\sin(\alpha + \beta) \\ \sin(\alpha + \beta) & \cos(\alpha + \beta) \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix}$$

$$= \begin{pmatrix} \cos \alpha \cos \beta - \sin \alpha \sin \beta & -\sin \alpha \cos \beta - \cos \alpha \sin \beta \\ \sin \alpha \cos \beta + \cos \alpha \sin \beta & \cos \alpha \cos \beta - \sin \alpha \sin \beta \end{pmatrix}$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) =$$

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$$\begin{pmatrix} \cos(\alpha + \beta) & -\sin(\alpha + \beta) \\ \sin(\alpha + \beta) & \cos(\alpha + \beta) \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix}$$

$$= \begin{pmatrix} \cos \alpha \cos \beta - \sin \alpha \sin \beta & -\sin \alpha \cos \beta - \cos \alpha \sin \beta \\ \sin \alpha \cos \beta + \cos \alpha \sin \beta & \cos \alpha \cos \beta - \sin \alpha \sin \beta \end{pmatrix}$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

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$$\begin{pmatrix} \cos(\alpha + \beta) & -\sin(\alpha + \beta) \\ \sin(\alpha + \beta) & \cos(\alpha + \beta) \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix}$$

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$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) =$$

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$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

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$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) =$$

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$$= \begin{pmatrix} \cos \alpha \cos \beta - \sin \alpha \sin \beta & -\sin \alpha \cos \beta - \cos \alpha \sin \beta \\ \sin \alpha \cos \beta + \cos \alpha \sin \beta & \cos \alpha \cos \beta - \sin \alpha \sin \beta \end{pmatrix}$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

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$$= \begin{pmatrix} \cos \alpha \cos \beta - \sin \alpha \sin \beta & -\sin \alpha \cos \beta - \cos \alpha \sin \beta \\ \sin \alpha \cos \beta + \cos \alpha \sin \beta & \cos \alpha \cos \beta - \sin \alpha \sin \beta \end{pmatrix}$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

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$$\tan(\alpha + \beta)$$

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$$\tan(\alpha + \beta) = \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)}$$

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$$\begin{aligned}\tan(\alpha + \beta) &= \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} \\ &= \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha \cos \beta - \sin \alpha \sin \beta}\end{aligned}$$

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$$\begin{aligned}
 \tan(\alpha + \beta) &= \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} \\
 &= \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha \cos \beta - \sin \alpha \sin \beta} \\
 &= \frac{\frac{\sin \alpha \cos \beta}{\cos \alpha \cos \beta} + \frac{\cos \alpha \sin \beta}{\cos \alpha \cos \beta}}{\frac{\cos \alpha \cos \beta}{\cos \alpha \cos \beta} - \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}}
 \end{aligned}$$

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$$\begin{aligned}\tan(\alpha + \beta) &= \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} \\ &= \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha \cos \beta - \sin \alpha \sin \beta} \\ &= \frac{\frac{\sin \alpha \cos \beta}{\cos \alpha \cos \beta} + \frac{\cos \alpha \sin \beta}{\cos \alpha \cos \beta}}{\frac{\cos \alpha \cos \beta}{\cos \alpha \cos \beta} - \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}} \\ &= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}\end{aligned}$$

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$$\begin{aligned}
 \tan(\alpha + \beta) &= \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} \\
 &= \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha \cos \beta - \sin \alpha \sin \beta} \\
 &= \frac{\frac{\sin \alpha \cos \beta}{\cos \alpha \cos \beta} + \frac{\cos \alpha \sin \beta}{\cos \alpha \cos \beta}}{\frac{\cos \alpha \cos \beta}{\cos \alpha \cos \beta} - \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}} \\
 &= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}
 \end{aligned}$$

$$\tan(\alpha + \beta)$$

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$$\begin{aligned}
 \tan(\alpha + \beta) &= \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} \\
 &= \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha \cos \beta - \sin \alpha \sin \beta} \\
 &= \frac{\frac{\sin \alpha \cos \beta}{\cos \alpha \cos \beta} + \frac{\cos \alpha \sin \beta}{\cos \alpha \cos \beta}}{\frac{\cos \alpha \cos \beta}{\cos \alpha \cos \beta} - \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}} \\
 &= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \\
 \tan(\alpha + \beta) &= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}
 \end{aligned}$$

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$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

Github:

<https://min7014.github.io/math20230416001.html>

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and you can see a picture moving.