대수적으로 이차부등식 풀기 ($ax^2 + bx + c < 0\ (a > 0,\ b, c \in \mathbb{R})$)

(Solving Quadratic Inequalities ($ax^2 + bx + c < 0\ (a > 0,\ b, c \in \mathbb{R})$) in Algebra)

Solving Quadratic Inequalities ($ax^2 + bx + c < 0$ ($a > 0$, $b, c \in \mathbb{R}$)) in Algebra

Start End

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$$ax^2 + bx + c < 0\ (a > 0,\ b, c \in \mathbb{R})$$

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Let $D = b^2 - 4ac$

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Let $D = b^2 - 4ac$

- $D > 0$: Let α and β be roots

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$ax^2 + bx + c < 0\ (a > 0,\ b, c \in \mathbb{R})$

Let $D = b^2 - 4ac$

- $D > 0$: Let α and β be roots of $ax^2 + bx + c = 0$

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Let $D = b^2 - 4ac$

- $D > 0$: Let α and β be roots of $ax^2 + bx + c = 0$ where $\alpha < \beta$.

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  $\therefore$

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  $\therefore \alpha < x$

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  $\therefore \alpha < x < \beta$ proof

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Let $D = b^2 - 4ac$

- $D > 0$: Let α and β be roots of $ax^2 + bx + c = 0$ where $\alpha < \beta$.
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- $D \leq 0$
  $\therefore$

Start End

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Let $D = b^2 - 4ac$

- $D > 0$: Let α and β be roots of $ax^2 + bx + c = 0$ where $\alpha < \beta$.
  $\therefore \alpha < x < \beta$ proof
- $D \leq 0$
  $\therefore$ No solutions proof

$$ax^2 + bx + c < 0$$

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$$
\begin{aligned}
ax^2 + bx + c &< 0 \quad (a > 0,\ b, c \in \mathbb{R}) \\
x^2 + \frac{b}{a}x + \frac{c}{a} &< 0 \quad (\because a > 0)
\end{aligned}
$$

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Let α and β be roots of $ax^2 + bx + c = 0$

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Let α and β be roots of $ax^2 + bx + c = 0$ where $\alpha < \beta$.

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Let α and β be roots of $ax^2 + bx + c = 0$ where $\alpha < \beta$.
$(\because b^2 - 4ac > 0)$

$$\begin{aligned} ax^2 + bx + c &< 0 \quad (a > 0,\ b, c \in \mathbb{R}) \\ x^2 + \frac{b}{a}x + \frac{c}{a} &< 0 \quad (\because a > 0) \end{aligned}$$

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$$(x - \alpha)(x - \beta) < 0$$

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Home Start End

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\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} &< 0
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Home Start End

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\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2} &< 0
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\end{aligned}
$$

$\therefore$ No solutions $(\because b^2 - 4ac \leq 0)$

Github:
https://min7014.github.io/math20210514001.html

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and you can see a picture moving.