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$$\exists p\ ,\ q \in \mathbb{Z} \ s.t.\ \alpha = \frac{q}{p} \text{ and } p|a_n \text{ and } q|a_0 \text{ and } gcd(p, q) = 1$$

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$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \ (a_n \neq 0 \ , \ a_i \in \mathbb{Z})$

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Start end

$$f(\alpha) \quad = \quad a_n\alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1\alpha + a_0$$

$$\text{Let } \alpha = \frac{q}{p} \; s.t. \; p, q \in \mathbb{Z} \text{ and } gcd(p,q) = 1 (\because \alpha \in \mathbb{Q})$$

$$f\Bigg($$

Start end

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$$f\left(\frac{q}{\right.$$

Start end

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Start end

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Start end

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Start end

$$
\begin{aligned}
f(\alpha) &= a_n\alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1\alpha + a_0 \\
&\text{Let } \alpha = \frac{q}{p}\ s.t.\ p, q \in \mathbb{Z} \text{ and } gcd(p, q) = 1(\because \alpha \in \mathbb{Q}) \\
f\left(\frac{q}{p}\right) &= a_n
\end{aligned}
$$

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$$
\begin{aligned}
f(\alpha) &= a_n \alpha^n + a_{n-1} \alpha^{n-1} + \cdots + a_1 \alpha + a_0 \\
&\text{Let } \alpha = \frac{q}{p} \ s.t.\ p, q \in \mathbb{Z} \text{ and } gcd(p, q) = 1 (\because \alpha \in \mathbb{Q}) \\
f\left(\frac{q}{p}\right) &= a_n \left(\frac{q}{p}\right)^n
\end{aligned}
$$

Start end

$$f(\alpha) = a_n\alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1\alpha + a_0$$

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$$f\left(\frac{q}{p}\right) = a_n\left(\frac{q}{p}\right)^n +$$

Start end

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f(\alpha) &= a_n \alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1\alpha + a_0 \\
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\end{aligned}
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Start end

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$$f\left(\frac{q}{p}\right) = a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1$$

Start end

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\end{aligned}
$$

▸ Start ▸ end

$$
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$$a_n \left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1 \frac{q}{p} + a_0 =$$

▸ Start ▸ end

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\end{aligned}
$$

$$a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0 = 0$$

▸ Start ▸ end

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a_n

▸ Start ▸ end

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$$a_n q^n$$

Start end

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$$a_n q^n +$$

Start end

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$$a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0 = 0$$

$$a_n q^n + a_{n-1}$$

▶ Start ▶ end

$$f(\alpha) = a_n\alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1\alpha + a_0$$

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$$a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0 = 0$$

$$a_n q^n + a_{n-1}q^{n-1}$$

Start end

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$$a_n \left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1 \frac{q}{p} + a_0 = 0$$

$$a_n q^n + a_{n-1} q^{n-1} p$$

Start end

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$$a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0 = 0$$

$$a_n q^n + a_{n-1}q^{n-1}p +$$

▸ Start ▸ end

$$\begin{aligned} f(\alpha) &= a_n \alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1\alpha + a_0 \\ & \quad \text{Let } \alpha = \frac{q}{p}\ s.t.\ p, q \in \mathbb{Z} \text{ and } gcd(p,q) = 1 (\because \alpha \in \mathbb{Q}) \\ f\left(\frac{q}{p}\right) &= a_n \left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1 \frac{q}{p} + a_0 \end{aligned}$$

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$$a_n q^n + a_{n-1} q^{n-1} p + \cdots$$

▸ Start ▸ end

$$
\begin{aligned}
f(\alpha) &= a_n\alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1\alpha + a_0 \\
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f\left(\frac{q}{p}\right) &= a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0
\end{aligned}
$$

$$a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0 = 0$$

$$a_n q^n + a_{n-1}q^{n-1}p + \cdots +$$

Start end

$$
\begin{aligned}
f(\alpha) &= a_n \alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1 \alpha + a_0 \\
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\end{aligned}
$$

$$
a_n \left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1 \frac{q}{p} + a_0 = 0
$$

$$
a_n q^n + a_{n-1} q^{n-1} p + \cdots + a_1
$$

Start end

$$f(\alpha) = a_n\alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1\alpha + a_0$$

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$$a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0 = 0$$

$$a_n q^n + a_{n-1}q^{n-1}p + \cdots + a_1 q$$

Start end

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$$a_n q^n + a_{n-1}q^{n-1}p + \cdots + a_1 q p^{n-1}$$

Start end

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$$a_n \left(\frac{q}{p}\right)^n + a_{n-1} \left(\frac{q}{p}\right)^{n-1} + \cdots + a_1 \frac{q}{p} + a_0 = 0$$

$$a_n q^n + a_{n-1} q^{n-1} p + \cdots + a_1 q p^{n-1} +$$

Start end

$$
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\end{aligned}
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Start end

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▸ Start ▸ end

$$\begin{aligned} f(\alpha) &= a_n\alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1\alpha + a_0 \\ &\quad \text{Let } \alpha = \frac{q}{p}\ s.t.\ p, q \in \mathbb{Z} \text{ and } gcd(p,q) = 1 (\because \alpha \in \mathbb{Q}) \\ f\left(\frac{q}{p}\right) &= a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0 \end{aligned}$$

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Start end

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$$a_n q^n + a_{n-1} q^{n-1} p + \cdots + a_1 q p^{n-1} + a_0 p^n = 0$$

Start end

$$\begin{aligned} f(\alpha) &= a_n\alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1\alpha + a_0 \\ &\quad \text{Let } \alpha = \frac{q}{p}\ s.t.\ p, q \in \mathbb{Z} \text{ and } gcd(p,q) = 1 (\because \alpha \in \mathbb{Q}) \\ f\left(\frac{q}{p}\right) &= a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0 \end{aligned}$$

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Start end

$$f(\alpha) = a_n \alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1\alpha + a_0$$

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$$f\left(\frac{q}{p}\right) = a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0$$

$$a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0 = 0$$

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$a_n q^n$

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$$\begin{aligned} f(\alpha) &= a_n \alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1 \alpha + a_0 \\ & \text{Let } \alpha = \frac{q}{p} \ s.t.\ p, q \in \mathbb{Z} \text{ and } gcd(p,q) = 1 (\because \alpha \in \mathbb{Q}) \\ f\left(\frac{q}{p}\right) &= a_n \left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1 \frac{q}{p} + a_0 \end{aligned}$$

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▸ Start ▸ end

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\end{aligned}
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\end{aligned}
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a_n \left(\frac{q}{p}\right)^n + a_{n-1} \left(\frac{q}{p}\right)^{n-1} + \cdots + a_1 \frac{q}{p} + a_0 = 0
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a_n q^n + a_{n-1} q^{n-1} p + \cdots + a_1 q p^{n-1} + a_0 p^n = 0
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\begin{aligned}
a_n q^n &= -a_{n-1} q^{n-1} p - \cdots - a_0 p^n \\
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\end{aligned}
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$$\begin{aligned} a_n q^n &= -a_{n-1} q^{n-1} p - \cdots - a_0 p^n \\ a_n q^n &= (- a_{n-1} \end{aligned}$$

▸ Start ▸ end

$$\begin{aligned} f(\alpha) &= a_n \alpha^n + a_{n-1} \alpha^{n-1} + \cdots + a_1 \alpha + a_0 \\ & \text{Let } \alpha = \frac{q}{p} \ s.t. \ p, q \in \mathbb{Z} \text{ and } gcd(p, q) = 1 (\because \alpha \in \mathbb{Q}) \\ f\left(\frac{q}{p}\right) &= a_n \left(\frac{q}{p}\right)^n + a_{n-1} \left(\frac{q}{p}\right)^{n-1} + \cdots + a_1 \frac{q}{p} + a_0 \end{aligned}$$

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\begin{aligned}
f(\alpha) &= a_n \alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1\alpha + a_0 \\
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a_n \left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1 \frac{q}{p} + a_0 = 0
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a_n q^n + a_{n-1} q^{n-1} p + \cdots + a_1 q p^{n-1} + a_0 p^n = 0
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\begin{aligned}
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a_n q^n &= (-a_{n-1} q^{n-1} -
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\begin{aligned}
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▸ Start ▸ end

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\end{aligned}
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a_n \left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1 \frac{q}{p} + a_0 = 0
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a_n q^n + a_{n-1} q^{n-1} p + \cdots + a_1 q p^{n-1} + a_0 p^n = 0
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\begin{aligned}
a_n q^n &= - a_{n-1} q^{n-1} p - \cdots - a_0 p^n \\
a_n q^n &= (- a_{n-1} q^{n-1} - \cdots - a_0 p^{n-1})
\end{aligned}
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\begin{aligned}
f(\alpha) &= a_n\alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1\alpha + a_0 \\
&\quad \text{Let } \alpha = \frac{q}{p} \ s.t.\ p, q \in \mathbb{Z} \text{ and } gcd(p,q) = 1(\because \alpha \in \mathbb{Q}) \\
f\left(\frac{q}{p}\right) &= a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0
\end{aligned}
$$

$$a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0 = 0$$

$$a_n q^n + a_{n-1}q^{n-1}p + \cdots + a_1 q p^{n-1} + a_0 p^n = 0$$

$$
\begin{aligned}
a_n q^n &= -a_{n-1}q^{n-1}p - \cdots - a_0 p^n \\
a_n q^n &= (-a_{n-1}q^{n-1} - \cdots - a_0 p^{n-1})p
\end{aligned}
$$

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$$
\begin{aligned}
f(\alpha) &= a_n \alpha^n + a_{n-1} \alpha^{n-1} + \cdots + a_1 \alpha + a_0 \\
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\end{aligned}
$$

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$$a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0 = 0$$

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$$\begin{aligned} f(\alpha) &= a_n\alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1\alpha + a_0 \\ &\quad \text{Let } \alpha = \frac{q}{p}\ s.t.\ p, q \in \mathbb{Z} \text{ and } gcd(p,q) = 1 (\because \alpha \in \mathbb{Q}) \\ f\left(\frac{q}{p}\right) &= a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0 \end{aligned}$$

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$$\begin{aligned} a_n q^n &= -a_{n-1}q^{n-1}p - \cdots - a_0 p^n \\ a_n q^n &= (-a_{n-1}q^{n-1} - \cdots - a_0 p^{n-1})p \qquad \Rightarrow \quad p | a_n \end{aligned}$$

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\begin{aligned}
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$$a_n \left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1 \frac{q}{p} + a_0 = 0$$

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& \quad (\because
\end{aligned}
$$

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$$f(\alpha) = a_n\alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1\alpha + a_0$$

Let $\alpha = \frac{q}{p}$ $s.t.$ $p, q \in \mathbb{Z}$ and $gcd(p,q) = 1(\because \alpha \in \mathbb{Q})$

$$f\left(\frac{q}{p}\right) = a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0$$

$$a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0 = 0$$

$$a_n q^n + a_{n-1}q^{n-1}p + \cdots + a_1 q p^{n-1} + a_0 p^n = 0$$

$$a_n q^n = -a_{n-1}q^{n-1}p - \cdots - a_0 p^n$$

$$a_n q^n = (-a_{n-1}q^{n-1} - \cdots - a_0 p^{n-1})p \quad \Rightarrow \quad p | a_n$$

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$$
\begin{aligned}
f(\alpha) &= a_n\alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1\alpha + a_0 \\
& \text{Let } \alpha = \frac{q}{p}\ s.t.\ p, q \in \mathbb{Z} \text{ and } gcd(p,q) = 1 (\because \alpha \in \mathbb{Q}) \\
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\end{aligned}
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$$a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0 = 0$$

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\end{aligned}
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a_n \left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1 \frac{q}{p} + a_0 = 0
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$$\begin{aligned} f(\alpha) &= a_n\alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1\alpha + a_0 \\ &\quad \text{Let } \alpha = \frac{q}{p}\ s.t.\ p, q \in \mathbb{Z} \text{ and } gcd(p,q) = 1 (\because \alpha \in \mathbb{Q}) \\ f\left(\frac{q}{p}\right) &= a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0 \end{aligned}$$

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a_0

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$$f(\alpha) = a_n\alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1\alpha + a_0$$

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$$f\left(\frac{q}{p}\right) = a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0$$

$$a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0 = 0$$

$$a_n q^n + a_{n-1}q^{n-1}p + \cdots + a_1 q p^{n-1} + a_0 p^n = 0$$

$$a_n q^n = -a_{n-1}q^{n-1}p - \cdots - a_0 p^n$$

$$a_n q^n = (-a_{n-1}q^{n-1} - \cdots - a_0 p^{n-1})p \quad \Rightarrow \quad p | a_n$$

$$(\because gcd(p, q) = 1, \text{Euclid's Lemma})$$

$$a_0 p^n$$

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$$\begin{aligned} f(\alpha) &= a_n\alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1\alpha + a_0 \\ &\quad \text{Let } \alpha = \frac{q}{p}\ s.t.\ p, q \in \mathbb{Z} \text{ and } gcd(p,q) = 1(\because \alpha \in \mathbb{Q}) \\ f\left(\frac{q}{p}\right) &= a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0 \end{aligned}$$

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\end{aligned}
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a_0 p^n &= -
\end{aligned}
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\end{aligned}
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$$
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$$

$$
\begin{aligned}
a_n q^n &= - a_{n-1} q^{n-1} p - \cdots - a_0 p^n \\
a_n q^n &= (- a_{n-1} q^{n-1} - \cdots - a_0 p^{n-1}) p \qquad \Rightarrow \qquad p | a_n \\
&\quad (\because gcd(p, q) = 1, \text{Euclid's Lemma}) \\
a_0 p^n &= - a_n q^n
\end{aligned}
$$

Start end

$$\begin{aligned} f(\alpha) &= a_n\alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1\alpha + a_0 \\ &\quad \text{Let } \alpha = \frac{q}{p}\ s.t.\ p, q \in \mathbb{Z} \text{ and } gcd(p,q) = 1 (\because \alpha \in \mathbb{Q}) \\ f\left(\frac{q}{p}\right) &= a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0 \end{aligned}$$

$$a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0 = 0$$

$$a_n q^n + a_{n-1}q^{n-1}p + \cdots + a_1 q p^{n-1} + a_0 p^n = 0$$

$$\begin{aligned} a_n q^n &= -a_{n-1}q^{n-1}p - \cdots - a_0 p^n \\ a_n q^n &= (-a_{n-1}q^{n-1} - \cdots - a_0 p^{n-1})p \qquad \Rightarrow \quad p | a_n \\ &\quad (\because gcd(p,q) = 1, \text{Euclid's Lemma}) \\ a_0 p^n &= -a_n q^n - \end{aligned}$$

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$$\begin{aligned} f(\alpha) &= a_n\alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1\alpha + a_0 \\ &\quad \text{Let } \alpha = \frac{q}{p}\ s.t.\ p, q \in \mathbb{Z} \text{ and } gcd(p, q) = 1(\because \alpha \in \mathbb{Q}) \\ f\left(\frac{q}{p}\right) &= a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0 \end{aligned}$$

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▸ Start ▸ end

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▸ Start ▸ end

$$f(\alpha) = a_n \alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1 \alpha + a_0$$

Let $\alpha = \dfrac{q}{p}$ $s.t.$ $p, q \in \mathbb{Z}$ and $gcd(p, q) = 1(\because \alpha \in \mathbb{Q})$

$$f\left(\frac{q}{p}\right) = a_n \left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1 \frac{q}{p} + a_0$$

$$a_n \left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1 \frac{q}{p} + a_0 = 0$$

$$a_n q^n + a_{n-1} q^{n-1} p + \cdots + a_1 q p^{n-1} + a_0 p^n = 0$$

$$a_n q^n = -a_{n-1} q^{n-1} p - \cdots - a_0 p^n$$

$$a_n q^n = (-a_{n-1} q^{n-1} - \cdots - a_0 p^{n-1}) p \quad \Rightarrow \quad p | a_n$$

$(\because gcd(p, q) = 1,$ Euclid's Lemma$)$

$$a_0 p^n = -a_n q^n - \cdots - a_1 q p^{n-1}$$

Start end

$$\begin{aligned} f(\alpha) &= a_n\alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1\alpha + a_0 \\ & \text{Let } \alpha = \frac{q}{p}\ s.t.\ p, q \in \mathbb{Z} \text{ and } gcd(p,q) = 1 (\because \alpha \in \mathbb{Q}) \\ f\left(\frac{q}{p}\right) &= a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0 \end{aligned}$$

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\end{aligned}$$

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▸ Start ▸ end

$$
\begin{aligned}
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a_0 p^n &= -a_n q^n - \cdots - a_1 q p^{n-1} \\
a_0 p^n &= q
\end{aligned}
$$

Start end

$$\begin{aligned} f(\alpha) &= a_n\alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1\alpha + a_0 \\ &\quad \text{Let } \alpha = \frac{q}{p}\ s.t.\ p, q \in \mathbb{Z} \text{ and } gcd(p,q) = 1(\because \alpha \in \mathbb{Q}) \\ f\left(\frac{q}{p}\right) &= a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0 \end{aligned}$$

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Start end

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▸ Start ▸ end

$$\begin{aligned} f(\alpha) &= a_n \alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1 \alpha + a_0 \\ &\quad \text{Let } \alpha = \frac{q}{p} \; s.t. \; p, q \in \mathbb{Z} \text{ and } gcd(p,q) = 1 (\because \alpha \in \mathbb{Q}) \\ f\left(\frac{q}{p}\right) &= a_n \left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1 \frac{q}{p} + a_0 \end{aligned}$$

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Start end

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\begin{aligned}
f(\alpha) &= a_n\alpha^n + a_{n-1}\alpha^{n-1} + \cdots + a_1\alpha + a_0 \\
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\end{aligned}
$$

$$
a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0 = 0
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$$
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▸ Start ▸ end

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Start end

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$$a_n\left(\frac{q}{p}\right)^n + a_{n-1}\left(\frac{q}{p}\right)^{n-1} + \cdots + a_1\frac{q}{p} + a_0 = 0$$

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\end{aligned}
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Github:
https://min7014.github.io/math20201221001.html

Click or paste URL into the URL search bar, and you can see a picture moving.