$$a \mid b$$

$a \mid b \ (a, b \in \mathbb{Z})$

$a \mid b \, (a, b \in \mathbb{Z}) \quad : \quad a$ divides b.

$a \mid b \; (a, b \in \mathbb{Z})$: a divides b.
a is a divisor of b.

$$a \mid b \; (a, b \in \mathbb{Z}) \quad : \quad \begin{array}{l} a \text{ divides } b. \\ a \text{ is a divisor of } b. \\ b \text{ is a multiple of } a. \end{array}$$

$a \mid b \ (a, b \in \mathbb{Z})$: a divides b.
a is a divisor of b.
b is a multiple of a.
$i.e.$

$a \mid b \ (a, b \in \mathbb{Z})$: a divides b.
a is a divisor of b.
b is a multiple of a.
$i.e. \ \exists k \in \mathbb{Z}$

$$a \mid b \; (a, b \in \mathbb{Z}) \quad : \quad a \text{ divides } b.$$

a is a divisor of b.

b is a multiple of a.

$i.e.\ \exists k \in \mathbb{Z}\ s.t.\ b = ak$

$$a \mid b \; (a, b \in \mathbb{Z}) \quad : \quad \begin{array}{l} a \text{ divides } b. \\ a \text{ is a divisor of } b. \\ b \text{ is a multiple of } a. \\ i.e.\ \exists k \in \mathbb{Z}\ s.t.\ b = ak \end{array}$$

- $a \mid b$

$a \mid b \ (a, b \in \mathbb{Z})$: a divides b.
a is a divisor of b.
b is a multiple of a.
$i.e. \ \exists k \in \mathbb{Z} \ s.t. \ b = ak$

- $a \mid b, \ b \mid c$

$$a \mid b \; (a, b \in \mathbb{Z}) \quad : \quad \begin{array}{l} a \text{ divides } b. \\ a \text{ is a divisor of } b. \\ b \text{ is a multiple of } a. \\ i.e. \; \exists k \in \mathbb{Z} \; s.t. \; b = ak \end{array}$$

- $a \mid b, \; b \mid c \Rightarrow a \mid c$ ▸ Proof

$$a \mid b \; (a, b \in \mathbb{Z}) \quad : \quad \begin{array}{l} a \text{ divides } b. \\ a \text{ is a divisor of } b. \\ b \text{ is a multiple of } a. \\ i.e.\ \exists k \in \mathbb{Z}\ s.t.\ b = ak \end{array}$$

- $a \mid b,\ b \mid c \Rightarrow a \mid c$ ▸ Proof

  $ex)\ 2 \mid 6$

$$
\begin{aligned}
a \mid b \; (a, b \in \mathbb{Z}) \quad : \quad & a \text{ divides } b. \\
& a \text{ is a divisor of } b. \\
& b \text{ is a multiple of } a. \\
& i.e.\ \exists k \in \mathbb{Z}\ s.t.\ b = ak
\end{aligned}
$$

- $a \mid b,\ b \mid c \Rightarrow a \mid c$ ▸ Proof

  $ex)\ 2 \mid 6,\ 6 \mid 18$

$$a \mid b \ (a, b \in \mathbb{Z}) \quad : \quad \begin{array}{l} a \text{ divides } b. \\ a \text{ is a divisor of } b. \\ b \text{ is a multiple of } a. \\ i.e.\ \exists k \in \mathbb{Z}\ s.t.\ b = ak \end{array}$$

- $a \mid b,\ b \mid c \Rightarrow a \mid c$ ▸ Proof

  $ex)\ 2 \mid 6,\ 6 \mid 18 \Rightarrow 2 \mid 18$

$$a \mid b \; (a, b \in \mathbb{Z}) \quad : \quad \begin{array}{l} a \text{ divides } b. \\ a \text{ is a divisor of } b. \\ b \text{ is a multiple of } a. \\ i.e. \ \exists k \in \mathbb{Z} \ s.t. \ b = ak \end{array}$$

- $a \mid b, \ b \mid c \Rightarrow a \mid c$ ▸ Proof
  $ex)\ 2 \mid 6, \ 6 \mid 18 \Rightarrow 2 \mid 18$

- $a \mid b$

$$a \mid b \ (a, b \in \mathbb{Z}) \quad : \quad \begin{array}{l} a \text{ divides } b. \\ a \text{ is a divisor of } b. \\ b \text{ is a multiple of } a. \\ i.e. \ \exists k \in \mathbb{Z} \ s.t. \ b = ak \end{array}$$

- $a \mid b, \ b \mid c \Rightarrow a \mid c$ ▸ Proof
  $ex) \ 2 \mid 6, \ 6 \mid 18 \Rightarrow 2 \mid 18$
- $a \mid b, b \mid a$

$$a \mid b \; (a, b \in \mathbb{Z}) \quad : \quad \begin{array}{l} a \text{ divides } b. \\ a \text{ is a divisor of } b. \\ b \text{ is a multiple of } a. \\ i.e. \; \exists k \in \mathbb{Z} \; s.t. \; b = ak \end{array}$$

- $a \mid b, \;\; b \mid c \Rightarrow a \mid c$ ▸ Proof
  $ex)\; 2 \mid 6, \;\; 6 \mid 18 \Rightarrow 2 \mid 18$
- $a \mid b, b \mid a \Rightarrow a = \pm b$ ▸ Proof

$$a \mid b \; (a, b \in \mathbb{Z}) \quad : \quad \begin{array}{l} a \text{ divides } b. \\ a \text{ is a divisor of } b. \\ b \text{ is a multiple of } a. \\ i.e. \; \exists k \in \mathbb{Z} \; s.t. \; b = ak \end{array}$$

- $a \mid b, \;\; b \mid c \Rightarrow a \mid c$ ▸ Proof
  $ex)\; 2 \mid 6, \;\; 6 \mid 18 \Rightarrow 2 \mid 18$
- $a \mid b, b \mid a \Rightarrow a = \pm b$ ▸ Proof
- $a \mid b$

$$a \mid b \; (a, b \in \mathbb{Z}) \quad : \quad \begin{array}{l} a \text{ divides } b. \\ a \text{ is a divisor of } b. \\ b \text{ is a multiple of } a. \\ i.e. \; \exists k \in \mathbb{Z} \; s.t. \; b = ak \end{array}$$

- $a \mid b, \;\; b \mid c \Rightarrow a \mid c$ ▸ Proof
  $ex) \; 2 \mid 6, \;\; 6 \mid 18 \Rightarrow 2 \mid 18$

- $a \mid b, b \mid a \Rightarrow a = \pm b$ ▸ Proof

- $a \mid b, a \mid c$

$$a \mid b \; (a, b \in \mathbb{Z}) \quad : \quad \begin{aligned} &a \text{ divides } b. \\ &a \text{ is a divisor of } b. \\ &b \text{ is a multiple of } a. \\ &i.e.\ \exists k \in \mathbb{Z}\ s.t.\ b = ak \end{aligned}$$

- $a \mid b,\ b \mid c \Rightarrow a \mid c$ ▸ Proof
  $ex)\ 2 \mid 6,\ 6 \mid 18 \Rightarrow 2 \mid 18$
- $a \mid b, b \mid a \Rightarrow a = \pm b$ ▸ Proof
- $a \mid b, a \mid c \Rightarrow a \mid (b \pm c)$ ▸ Proof

$$a \mid b \ (a, b \in \mathbb{Z}) \quad : \quad a \text{ divides } b.$$
a is a divisor of b.
b is a multiple of a.
$i.e.\ \exists k \in \mathbb{Z}\ s.t.\ b = ak$

- $a \mid b,\ \ b \mid c \Rightarrow a \mid c$ ▸ Proof
  $ex)\ 2 \mid 6,\ \ 6 \mid 18 \Rightarrow 2 \mid 18$

- $a \mid b, b \mid a \Rightarrow a = \pm b$ ▸ Proof

- $a \mid b, a \mid c \Rightarrow a \mid (b \pm c)$ ▸ Proof
  $ex)\ 3 \mid 6$

$$a \mid b \; (a, b \in \mathbb{Z}) \quad : \quad \begin{array}{l} a \text{ divides } b. \\ a \text{ is a divisor of } b. \\ b \text{ is a multiple of } a. \\ i.e.\ \exists k \in \mathbb{Z}\ s.t.\ b = ak \end{array}$$

- $a \mid b, \ b \mid c \Rightarrow a \mid c$ Proof
  $ex)\ 2 \mid 6, \ 6 \mid 18 \Rightarrow 2 \mid 18$
- $a \mid b, b \mid a \Rightarrow a = \pm b$ Proof
- $a \mid b, a \mid c \Rightarrow a \mid (b \pm c)$ Proof
  $ex)\ 3 \mid 6, 3 \mid 21$

$$a \mid b \; (a, b \in \mathbb{Z}) \quad : \quad \begin{array}{l} a \text{ divides } b. \\ a \text{ is a divisor of } b. \\ b \text{ is a multiple of } a. \\ i.e.\ \exists k \in \mathbb{Z} \ s.t.\ b = ak \end{array}$$

- $a \mid b, \ b \mid c \Rightarrow a \mid c$ Proof
  $ex)\ 2 \mid 6, \ 6 \mid 18 \Rightarrow 2 \mid 18$

- $a \mid b, b \mid a \Rightarrow a = \pm b$ Proof

- $a \mid b, a \mid c \Rightarrow a \mid (b \pm c)$ Proof
  $ex)\ 3 \mid 6, 3 \mid 21 \Rightarrow 3 \mid (6 \pm 21)$

Start

- $a \mid b,\ b \mid c \Rightarrow a \mid c$

▸ Start

- $a \mid b,\ b \mid c \Rightarrow a \mid c$

$$b = ak_1$$

Start

- $a \mid b,\ b \mid c \Rightarrow a \mid c$

  $b = ak_1, c = bk_2$

Start

- $a \mid b,\ b \mid c \Rightarrow a \mid c$

$$b = ak_1, c = bk_2\ (k_1, k_2 \in \mathbb{Z})$$

Start

- $a \mid b,\ b \mid c \Rightarrow a \mid c$

$$b = ak_1, c = bk_2 \ (k_1, k_2 \in \mathbb{Z})$$
$$c = (ak_1)k_2$$

Start

- $a \mid b,\ b \mid c \Rightarrow a \mid c$

$$b = ak_1, c = bk_2\ (k_1, k_2 \in \mathbb{Z})$$
$$c = (ak_1)k_2 = a(k_1k_2)$$

Start

- $a \mid b,\ b \mid c \Rightarrow a \mid c$

$$b = ak_1, c = bk_2\ (k_1, k_2 \in \mathbb{Z})$$
$$c = (ak_1)k_2 = a(k_1 k_2)$$
Let $k_3 = k_1 k_2$

Start

- $a \mid b,\ b \mid c \Rightarrow a \mid c$

$$b = ak_1, c = bk_2\ (k_1, k_2 \in \mathbb{Z})$$
$$c = (ak_1)k_2 = a(k_1k_2)$$
$$\text{Let } k_3 = k_1k_2$$
$$c = ak_3$$

Start

- $a \mid b,\ b \mid c \Rightarrow a \mid c$

$$
\begin{aligned}
& b = ak_1, c = bk_2\ (k_1, k_2 \in \mathbb{Z}) \\
& c = (ak_1)k_2 = a(k_1k_2) \\
& \text{Let } k_3 = k_1k_2 \\
& c = ak_3 \\
& \therefore a \mid c
\end{aligned}
$$

▸ Start

- $a \mid b, b \mid a \Rightarrow a = \pm b$

Start

- $a \mid b, b \mid a \Rightarrow a = \pm b$

$$b = ak_1$$

Start

- $a \mid b, b \mid a \Rightarrow a = \pm b$

  $b = ak_1, a = bk_2$

Start

- $a \mid b, b \mid a \Rightarrow a = \pm b$

$$b = ak_1, a = bk_2 \ (k_1, k_2 \in \mathbb{Z})$$

▸ Start

- $a \mid b, b \mid a \Rightarrow a = \pm b$

$$b = ak_1, a = bk_2 \; (k_1, k_2 \in \mathbb{Z})$$
$$a = (ak_1)k_2$$

Start

- $a \mid b, b \mid a \Rightarrow a = \pm b$

$$b = ak_1, a = bk_2 \ (k_1, k_2 \in \mathbb{Z})$$
$$a = (ak_1)k_2 = ak_1k_2$$

Start

- $a \mid b, b \mid a \Rightarrow a = \pm b$

$$b = ak_1, a = bk_2 \ (k_1, k_2 \in \mathbb{Z})$$
$$a = (ak_1)k_2 = ak_1k_2$$
$$a = ak_1k_2$$

▸ Start

- $a \mid b, b \mid a \Rightarrow a = \pm b$

$$b = ak_1, a = bk_2 \ (k_1, k_2 \in \mathbb{Z})$$
$$a = (ak_1)k_2 = ak_1k_2$$
$$a = ak_1k_2$$
$$a(k_1k_2 - 1) = 0$$

▸ Start

- $a \mid b, b \mid a \Rightarrow a = \pm b$

$$b = ak_1, a = bk_2 \ (k_1, k_2 \in \mathbb{Z})$$
$$a = (ak_1)k_2 = ak_1k_2$$
$$a = ak_1k_2$$
$$a(k_1k_2 - 1) = 0$$
$$a = 0 \text{ or } k_1k_2 = 1$$

▸ Start

- $a \mid b, b \mid a \Rightarrow a = \pm b$

$$
\begin{aligned}
& b = ak_1, a = bk_2 \ (k_1, k_2 \in \mathbb{Z}) \\
& a = (ak_1)k_2 = ak_1k_2 \\
& a = ak_1k_2 \\
& a(k_1k_2 - 1) = 0 \\
& a = 0 \text{ or } k_1k_2 = 1 \\
& (a = 0 \text{ and } b = 0) \text{ or } k_1 = \pm 1
\end{aligned}
$$

Start

- $a \mid b, b \mid a \Rightarrow a = \pm b$

$$
\begin{aligned}
&b = ak_1, a = bk_2 \ (k_1, k_2 \in \mathbb{Z}) \\
&a = (ak_1)k_2 = ak_1k_2 \\
&a = ak_1k_2 \\
&a(k_1k_2 - 1) = 0 \\
&a = 0 \text{ or } k_1k_2 = 1 \\
&(a = 0 \text{ and } b = 0) \text{ or } k_1 = \pm 1 (\because k_1, k_2 \in \mathbb{Z})
\end{aligned}
$$

Start

- $a \mid b, b \mid a \Rightarrow a = \pm b$

$$
\begin{aligned}
& b = ak_1, a = bk_2 \ (k_1, k_2 \in \mathbb{Z}) \\
& a = (ak_1)k_2 = ak_1k_2 \\
& a = ak_1k_2 \\
& a(k_1k_2 - 1) = 0 \\
& a = 0 \text{ or } k_1k_2 = 1 \\
& (a = 0 \text{ and } b = 0) \text{ or } k_1 = \pm 1 (\because k_1, k_2 \in \mathbb{Z}) \\
& (a = 0 \text{ and } b = 0) \text{ or } a = \pm b
\end{aligned}
$$

Start

- $a \mid b, b \mid a \Rightarrow a = \pm b$

$$
\begin{aligned}
&b = ak_1, a = bk_2 \ (k_1, k_2 \in \mathbb{Z}) \\
&a = (ak_1)k_2 = ak_1k_2 \\
&a = ak_1k_2 \\
&a(k_1k_2 - 1) = 0 \\
&a = 0 \text{ or } k_1k_2 = 1 \\
&(a = 0 \text{ and } b = 0) \text{ or } k_1 = \pm 1(\because k_1, k_2 \in \mathbb{Z}) \\
&(a = 0 \text{ and } b = 0) \text{ or } a = \pm b \\
&\therefore a = \pm b
\end{aligned}
$$

▶ Start

- $a \mid b, a \mid c \Rightarrow a \mid (b \pm c)$

Start

- $a \mid b, a \mid c \Rightarrow a \mid (b \pm c)$

  $b = ak_1$

Start

- $a \mid b, a \mid c \Rightarrow a \mid (b \pm c)$

$$b = ak_1, c = ak_2$$

▸ Start

- $a \mid b, a \mid c \Rightarrow a \mid (b \pm c)$

$$b = ak_1, c = ak_2 \; (k_1, k_2 \in \mathbb{Z})$$

Start

- $a \mid b, a \mid c \Rightarrow a \mid (b \pm c)$

$$b = ak_1, c = ak_2 \ (k_1, k_2 \in \mathbb{Z})$$
$$b \pm c = a(k_1 \pm k_2)$$

Start

- $a \mid b, a \mid c \Rightarrow a \mid (b \pm c)$

$$b = ak_1, c = ak_2 \ (k_1, k_2 \in \mathbb{Z})$$
$$b \pm c = a(k_1 \pm k_2)$$
$$\text{Let } k_3 = k_1 \pm k_2$$

Start

- $a \mid b, a \mid c \Rightarrow a \mid (b \pm c)$

$$
\begin{aligned}
&b = ak_1, c = ak_2 \ (k_1, k_2 \in \mathbb{Z}) \\
&b \pm c = a(k_1 \pm k_2) \\
&\text{Let } k_3 = k_1 \pm k_2 \\
&b \pm c = ak_3
\end{aligned}
$$

Start

- $a \mid b, a \mid c \Rightarrow a \mid (b \pm c)$

$$b = ak_1, c = ak_2 \ (k_1, k_2 \in \mathbb{Z})$$
$$b \pm c = a(k_1 \pm k_2)$$
$$\text{Let } k_3 = k_1 \pm k_2$$
$$b \pm c = ak_3$$
$$\therefore a \mid (b \pm c)$$

Github:
https://min7014.github.io/math20201209001.html

Click or paste URL into the URL search bar, and you can see a picture moving.