정수 나누기 계산법
(The Integer Division Algorithm)

The Integer Division Algorithm

$$\forall A, \forall B (\neq 0) \in \mathbb{Z}$$

$$\forall A, \forall B (\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z}$$

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t.$$

$$\forall A, \forall B (\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R$$

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] ▸ Proof

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

[Existence] ▸ Proof [Uniqueness] ▸ Proof

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] Proof [Uniqueness] Proof

$ex) \;\; 7$

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] Proof [Uniqueness] Proof

$ex) \;\; 7 \quad =$

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] Proof [Uniqueness] Proof

$ex) \;\; 7 \;\; = \qquad 2$

$$\forall A, \forall B (\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

[Existence] Proof [Uniqueness] Proof

$$ex) \;\; 7 \;\; = \;\; 2 \;\; \times$$

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] Proof [Uniqueness] Proof

$ex) \;\; 7 \;\; = \;\; 2 \;\; \times \;\; 3$

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] Proof [Uniqueness] Proof

$ex) \;\; 7 \;\; = \;\; 2 \;\; \times \;\; 3 \;\; +$

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

[Existence] ▸ Proof [Uniqueness] ▸ Proof

$$ex) \;\; 7 \;\; = \;\; 2 \;\; \times \;\; 3 \;\; + \;\; 1$$

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

[Existence] ▸ Proof [Uniqueness] ▸ Proof

$$ex) \; 7 = 2 \times 3 + 1 \; , \; 0 \leq 1 < |2|$$

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] Proof [Uniqueness] Proof

$ex) \;\; 7 = 2 \times 3 + 1 \;\;, \;\; 0 \leq 1 < |2|$

7

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] ▸ Proof [Uniqueness] ▸ Proof

$$ex) \;\; 7 = 2 \times 3 + 1 \;, \;\; 0 \leq 1 < |2|$$

$$7 =$$

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

[Existence] ▸ Proof [Uniqueness] ▸ Proof

$$ex) \; 7 = 2 \times 3 + 1 \;, \; 0 \leq 1 < |2|$$

$$7 = (-2)$$

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] ▸ Proof [Uniqueness] ▸ Proof

$ex)\; 7 = 2 \times 3 + 1 \;,\; 0 \leq 1 < |2|$

$7 = (-2) \times$

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

[Existence] ▸ Proof [Uniqueness] ▸ Proof

$$ex) \;\; 7 = 2 \times 3 + 1 \; , \; 0 \leq 1 < |2|$$

$$7 = (-2) \times (-3)$$

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence] ▸ Proof [Uniqueness] ▸ Proof

$$ex) \; 7 = 2 \times 3 + 1 \; , \; 0 \leq 1 < |2|$$

$$7 = (-2) \times (-3) +$$

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

[Existence] Proof [Uniqueness] Proof

$$ex) \; 7 = 2 \times 3 + 1 \; , \; 0 \leq 1 < |2|$$

$$7 = (-2) \times (-3) + 1$$

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

[Existence] ▸ Proof [Uniqueness] ▸ Proof

$$ex) \; 7 = 2 \times 3 + 1 \;, \; 0 \leq 1 < |2|$$

$$7 = (-2) \times (-3) + 1 \;, \; 0 \leq 1 < |-2|$$

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \ \ s.t. \ \ A = BQ + R, \ \ 0 \leq R < |B|$$

[Existence] Proof [Uniqueness] Proof

$$ex) \ 7 = 2 \times 3 + 1 \ , \ 0 \leq 1 < |2|$$

$$7 = (-2) \times (-3) + 1 \ , \ 0 \leq 1 < |-2|$$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence]

Start

$\forall A, \forall B (\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence]

Let

Start

$\forall A, \forall B (\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence]

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

▸ Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence]

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

$\exists R \in S$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence]

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

$\exists R \in S \;\; s.t.$

▸ Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence]

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S$

▸ Start

$\forall A, \forall B (\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \ \ s.t. \ \ A = BQ + R, \ \ 0 \leq R < |B|$

[Existence]

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \quad \cdots\cdots (1)$

$\exists R \in S \ \ s.t. \ \ x \in S \Rightarrow$

▸ Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \ \ s.t. \ \ A = BQ + R, \ \ 0 \leq R < |B|$

[Existence]

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \ \ \cdots\cdots (1)$

$\exists R \in S \ \ s.t. \ \ x \in S \Rightarrow R \leq x$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

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$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \;\; (\because S \subset \mathbb{N} \cup \{0\})$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence]

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \;\; (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z}$

Start

$\forall A, \forall B (\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \ \ s.t. \ \ A = BQ + R, \ \ 0 \leq R < |B|$

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$\exists Q \in \mathbb{Z} \ \ s.t.$

▸ Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

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$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \;\; (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \;\; \cdots\cdots (2)$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence]

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Assume

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

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Assume $R \geq |B|$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

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Assume $R \geq |B|$

$A - BQ \geq |B|$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

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$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \;\; \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B| \;\; , \;\; A - BQ - |B| \geq 0$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

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$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \;\; \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B| \;\; , \;\; A - BQ - |B| \geq 0$

$A - B\left(Q + \frac{|B|}{B}\right)$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence]

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \quad \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \quad (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \quad \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B| \;\; , \;\; A - BQ - |B| \geq 0$

$A - B\left(Q + \frac{|B|}{B}\right) =$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \ \ s.t. \ \ A = BQ + R, \ \ 0 \leq R < |B|$

[Existence]

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \ \ \cdots\cdots (1)$

$\exists R \in S \ \ s.t. \ \ x \in S \Rightarrow R \leq x \ \ (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \ \ s.t. \ \ R = A - BQ \ \ \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B| \ \ , \ A - BQ - |B| \geq 0$

$A - B\left(Q + \frac{|B|}{B}\right) = A - BQ - |B|$

Start

$\forall A, \forall B (\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence]

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \;\; (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \;\; \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B| \;\; , \; A - BQ - |B| \geq 0$

$A - B\left(Q + \frac{|B|}{B}\right) = A - BQ - |B| <$

▸ Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence]

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \;\; (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \;\; \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B| \;\; , \;\; A - BQ - |B| \geq 0$

$$A - B\left(Q + \frac{|B|}{B}\right) = A - BQ - |B| < A - BQ$$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence]

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \quad \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \quad (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \quad \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B|$, $A - BQ - |B| \geq 0$

$$A - B\left(Q + \frac{|B|}{B}\right) = A - BQ - |B| < A - BQ \quad \therefore contradiction$$

Start

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

[Existence]

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \;\; (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \;\; \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B| \;\; , \;\; A - BQ - |B| \geq 0$

$$A - B\left(Q + \frac{|B|}{B}\right) = A - BQ - |B| < A - BQ \quad \therefore contradiction$$

$\therefore R < |B| \;\; \cdots\cdots (3)$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence]

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \;\; (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \;\; \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B| \;\; , \;\; A - BQ - |B| \geq 0$

$$A - B\left(Q + \frac{|B|}{B}\right) = A - BQ - |B| < A - BQ \quad \therefore \textit{contradiction}$$

$\therefore R < |B| \;\; \cdots\cdots (3)$

By (1), (2), (3)

$\therefore$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence]

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \;\; (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \;\; \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B| \;\; , \;\; A - BQ - |B| \geq 0$

$A - B\left(Q + \frac{|B|}{B}\right) = A - BQ - |B| < A - BQ \quad \therefore \textit{contradiction}$

$\therefore R < |B| \;\; \cdots\cdots (3)$

By (1), (2), (3)

$\therefore \exists Q, \exists R \in \mathbb{Z}$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Existence]

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \;\; \cdots\cdots (1)$

$\exists R \in S \;\; s.t. \;\; x \in S \Rightarrow R \leq x \;\; (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \;\; s.t. \;\; R = A - BQ \;\; \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B| \;\; , \;\; A - BQ - |B| \geq 0$

$A - B\left(Q + \frac{|B|}{B}\right) = A - BQ - |B| < A - BQ \quad \therefore contradiction$

$\therefore R < |B| \;\; \cdots\cdots (3)$

By (1), (2), (3)

$\therefore \exists Q, \exists R \in \mathbb{Z} \;\; s.t.$

▸ Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \ \ s.t. \ \ A = BQ + R, \ \ 0 \leq R < |B|$

[Existence]

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \ \ \cdots\cdots (1)$

$\exists R \in S \ \ s.t. \ \ x \in S \Rightarrow R \leq x \ \ (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \ \ s.t. \ \ R = A - BQ \ \ \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B| \ \ , \ A - BQ - |B| \geq 0$

$A - B\left(Q + \frac{|B|}{B}\right) = A - BQ - |B| < A - BQ \quad \therefore contradiction$

$\therefore R < |B| \ \ \cdots\cdots (3)$

By (1), (2), (3)

$\therefore \exists Q, \exists R \in \mathbb{Z} \ \ s.t. \ \ A = BQ + R,$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \ \ s.t. \ \ A = BQ + R, \ \ 0 \leq R < |B|$

[Existence]

Let $S = \{x | x = A - B \times n \geq 0, n \in \mathbb{Z}\} \ \ \cdots\cdots (1)$

$\exists R \in S \ \ s.t. \ \ x \in S \Rightarrow R \leq x \ \ (\because S \subset \mathbb{N} \cup \{0\})$

$\exists Q \in \mathbb{Z} \ \ s.t. \ \ R = A - BQ \ \ \cdots\cdots (2)$

Assume $R \geq |B|$

$A - BQ \geq |B| \ , \ A - BQ - |B| \geq 0$

$A - B\left(Q + \frac{|B|}{B}\right) = A - BQ - |B| < A - BQ \quad \therefore contradiction$

$\therefore R < |B| \ \ \cdots\cdots (3)$

By (1), (2), (3)

$\therefore \exists Q, \exists R \in \mathbb{Z} \ \ s.t. \ \ A = BQ + R, \ \ 0 \leq R < |B|$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \ \ s.t. \ \ A = BQ + R, \ \ 0 \leq R < |B|$

[Uniquness]

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \ \ s.t. \ \ A = BQ + R, \ \ 0 \leq R < |B|$

[Uniquness]

$Let \ A = BQ_1 + R_1, \ \ 0 \leq R_1 < |B|$

Start

$\forall A, \forall B (\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Uniquness]

$Let\ A = BQ_1 + R_1, \;\; 0 \leq R_1 < |B|$

$Let\ A = BQ_2 + R_2, \;\; 0 \leq R_2 < |B|$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Uniquness]

$Let\ A = BQ_1 + R_1, \;\; 0 \leq R_1 < |B|$

$Let\ A = BQ_2 + R_2, \;\; 0 \leq R_2 < |B|$

$-|B| < R_2 - R_1 < |B|$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Uniquness]

$Let\ A = BQ_1 + R_1, \;\; 0 \leq R_1 < |B|$

$Let\ A = BQ_2 + R_2, \;\; 0 \leq R_2 < |B|$

$-|B| < R_2 - R_1 < |B| \;\; , \;\; |R_2 - R_1| < |B| \;\; \cdots\cdots (1)$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Uniquness]

$Let\ A = BQ_1 + R_1, \;\; 0 \leq R_1 < |B|$

$Let\ A = BQ_2 + R_2, \;\; 0 \leq R_2 < |B|$

$-|B| < R_2 - R_1 < |B| \;\; , \;\; |R_2 - R_1| < |B| \;\; \cdots\cdots (1)$

$BQ_1 + R_1 = BQ_2 + R_2$

Start

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

[Uniquness]

$$Let \; A = BQ_1 + R_1, \;\; 0 \leq R_1 < |B|$$

$$Let \; A = BQ_2 + R_2, \;\; 0 \leq R_2 < |B|$$

$$-|B| < R_2 - R_1 < |B| \;\; , \;\; |R_2 - R_1| < |B| \;\; \cdots\cdots (1)$$

$$BQ_1 + R_1 = BQ_2 + R_2 \;\; , \;\; B(Q_1 - Q_2) = R_2 - R_1 \;\; \cdots\cdots (2)$$

Start

$\forall A, \forall B (\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Uniquness]

$Let \; A = BQ_1 + R_1, \;\; 0 \leq R_1 < |B|$

$Let \; A = BQ_2 + R_2, \;\; 0 \leq R_2 < |B|$

$-|B| < R_2 - R_1 < |B| \;\; , \;\; |R_2 - R_1| < |B| \;\; \cdots\cdots (1)$

$BQ_1 + R_1 = BQ_2 + R_2 \;\; , \;\; B(Q_1 - Q_2) = R_2 - R_1 \;\; \cdots\cdots (2)$

$by \; (1), (2)$

▸ Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \ \ s.t. \ \ A = BQ + R, \ \ 0 \leq R < |B|$

[Uniquness]

$Let\ A = BQ_1 + R_1, \ \ 0 \leq R_1 < |B|$

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$-|B| < R_2 - R_1 < |B| \ \ , \ \ |R_2 - R_1| < |B| \ \ \cdots\cdots (1)$

$BQ_1 + R_1 = BQ_2 + R_2 \ \ , \ \ B(Q_1 - Q_2) = R_2 - R_1 \ \ \cdots\cdots (2)$

$by\ (1), (2)$

$|B(Q_1 - Q_2)| = |R_2 - R_1| < |B|$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Uniquness]

$Let \; A = BQ_1 + R_1, \;\; 0 \leq R_1 < |B|$

$Let \; A = BQ_2 + R_2, \;\; 0 \leq R_2 < |B|$

$-|B| < R_2 - R_1 < |B| \;\; , \;\; |R_2 - R_1| < |B| \;\; \cdots\cdots(1)$

$BQ_1 + R_1 = BQ_2 + R_2 \;\; , \;\; B(Q_1 - Q_2) = R_2 - R_1 \;\; \cdots\cdots(2)$

$by \; (1), (2)$

$|B(Q_1 - Q_2)| = |R_2 - R_1| < |B|$

$|B||Q_1 - Q_2| < |B|$

Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z}\ \ s.t.\ \ A = BQ + R,\ \ 0 \leq R < |B|$

[Uniquness]

$Let\ A = BQ_1 + R_1,\ \ 0 \leq R_1 < |B|$

$Let\ A = BQ_2 + R_2,\ \ 0 \leq R_2 < |B|$

$-|B| < R_2 - R_1 < |B|\ \ ,\ \ |R_2 - R_1| < |B|\ \ \cdots\cdots(1)$

$BQ_1 + R_1 = BQ_2 + R_2\ \ ,\ \ B(Q_1 - Q_2) = R_2 - R_1\ \ \cdots\cdots(2)$

$by\ (1),(2)$

$|B(Q_1 - Q_2)| = |R_2 - R_1| < |B|$

$|B||Q_1 - Q_2| < |B|\ \ ,\ \ |Q_1 - Q_2| < 1$

▸ Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Uniquness]

$Let\ A = BQ_1 + R_1, \;\; 0 \leq R_1 < |B|$

$Let\ A = BQ_2 + R_2, \;\; 0 \leq R_2 < |B|$

$-|B| < R_2 - R_1 < |B| \;\; , \;\; |R_2 - R_1| < |B| \;\; \cdots\cdots (1)$

$BQ_1 + R_1 = BQ_2 + R_2 \;\; , \;\; B(Q_1 - Q_2) = R_2 - R_1 \;\; \cdots\cdots (2)$

$by\ (1), (2)$

$|B(Q_1 - Q_2)| = |R_2 - R_1| < |B|$

$|B||Q_1 - Q_2| < |B| \;\; , \;\; |Q_1 - Q_2| < 1$

$|Q_1 - Q_2| = 0$

▸ Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z}\ \ s.t.\ \ A = BQ + R,\ \ 0 \leq R < |B|$

[Uniquness]

$Let\ A = BQ_1 + R_1,\ \ 0 \leq R_1 < |B|$

$Let\ A = BQ_2 + R_2,\ \ 0 \leq R_2 < |B|$

$-|B| < R_2 - R_1 < |B|\ \ ,\ \ |R_2 - R_1| < |B|\ \ \cdots\cdots(1)$

$BQ_1 + R_1 = BQ_2 + R_2\ \ ,\ \ B(Q_1 - Q_2) = R_2 - R_1\ \ \cdots\cdots(2)$

$by\ (1), (2)$

$|B(Q_1 - Q_2)| = |R_2 - R_1| < |B|$

$|B||Q_1 - Q_2| < |B|\ \ ,\ \ |Q_1 - Q_2| < 1$

$|Q_1 - Q_2| = 0\ \ (\because Q_1, Q_2 \in \mathbb{Z})$

▸ Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z}\ \ s.t.\ \ A = BQ + R,\ \ 0 \leq R < |B|$

[Uniquness]

$Let\ A = BQ_1 + R_1,\ \ 0 \leq R_1 < |B|$

$Let\ A = BQ_2 + R_2,\ \ 0 \leq R_2 < |B|$

$-|B| < R_2 - R_1 < |B|\ \ ,\ \ |R_2 - R_1| < |B|\ \ \cdots\cdots (1)$

$BQ_1 + R_1 = BQ_2 + R_2\ \ ,\ \ B(Q_1 - Q_2) = R_2 - R_1\ \ \cdots\cdots (2)$

$by\ (1), (2)$

$|B(Q_1 - Q_2)| = |R_2 - R_1| < |B|$

$|B||Q_1 - Q_2| < |B|\ \ ,\ \ |Q_1 - Q_2| < 1$

$|Q_1 - Q_2| = 0\ \ (\because Q_1, Q_2 \in \mathbb{Z})$

$\therefore$

▸ Start

$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$

[Uniquness]

$Let\ A = BQ_1 + R_1, \;\; 0 \leq R_1 < |B|$

$Let\ A = BQ_2 + R_2, \;\; 0 \leq R_2 < |B|$

$-|B| < R_2 - R_1 < |B| \;\;, \;\; |R_2 - R_1| < |B| \;\; \cdots\cdots (1)$

$BQ_1 + R_1 = BQ_2 + R_2 \;\;, \;\; B(Q_1 - Q_2) = R_2 - R_1 \;\; \cdots\cdots (2)$

$by\ (1), (2)$

$|B(Q_1 - Q_2)| = |R_2 - R_1| < |B|$

$|B||Q_1 - Q_2| < |B| \;\;, \;\; |Q_1 - Q_2| < 1$

$|Q_1 - Q_2| = 0 \;\; (\because Q_1, Q_2 \in \mathbb{Z})$

$\therefore \;\; Q_1 = Q_2$

Start

$$\forall A, \forall B(\neq 0) \in \mathbb{Z}, \exists! Q, \exists! R \in \mathbb{Z} \;\; s.t. \;\; A = BQ + R, \;\; 0 \leq R < |B|$$

[Uniqusness]

$$Let\ A = BQ_1 + R_1, \;\; 0 \leq R_1 < |B|$$

$$Let\ A = BQ_2 + R_2, \;\; 0 \leq R_2 < |B|$$

$$-|B| < R_2 - R_1 < |B| \;\;,\;\; |R_2 - R_1| < |B| \;\;\cdots\cdots(1)$$

$$BQ_1 + R_1 = BQ_2 + R_2 \;\;,\;\; B(Q_1 - Q_2) = R_2 - R_1 \;\;\cdots\cdots(2)$$

$$by\ (1),(2)$$

$$|B(Q_1 - Q_2)| = |R_2 - R_1| < |B|$$

$$|B||Q_1 - Q_2| < |B| \;\;,\;\; |Q_1 - Q_2| < 1$$

$$|Q_1 - Q_2| = 0 \;\; (\because Q_1, Q_2 \in \mathbb{Z})$$

$$\therefore \;\; Q_1 = Q_2 \;\;,\;\; R_1 = R_2$$

Github:
https://min7014.github.io/math20201204001.html

Click or paste URL into the URL search bar, and you can see a picture moving.