$$(x+y+z)^3-x^3-y^3-z^3 = 3(x+y)(y+z)(z+x)$$

$$(x+y+z)^3 - x^3 - y^3 - z^3 = 3(x+y)(y+z)(z+x)$$

$$(x+y+z)^3 - x^3 - y^3 - z^3$$

$$
\begin{aligned}
&(x+y+z)^3 - x^3 - y^3 - z^3 \\
=&\{(x+y)+z\}^3 - x^3 - y^3 - z^3
\end{aligned}
$$

$$
\begin{aligned}
& (x+y+z)^3 - x^3 - y^3 - z^3 \\
=& \{(x+y)+z\}^3 - x^3 - y^3 - z^3 \\
=& (x+y)^3 + z^3 + 3(x+y)z\{(x+y)+z\} - x^3 - y^3 - z^3
\end{aligned}
$$

$$
\begin{aligned}
& (x+y+z)^3 - x^3 - y^3 - z^3 \\
=& \{(x+y)+z\}^3 - x^3 - y^3 - z^3 \\
=& (x+y)^3 + z^3 + 3(x+y)z\{(x+y)+z\} - x^3 - y^3 - z^3 \\
=& (x+y)^3 + z^3 + 3(x+y)z(x+y+z) - x^3 - y^3 - z^3
\end{aligned}
$$

$$
\begin{aligned}
& (x+y+z)^3 - x^3 - y^3 - z^3 \\
=& \{(x+y)+z\}^3 - x^3 - y^3 - z^3 \\
=& (x+y)^3 + z^3 + 3(x+y)z\{(x+y)+z\} - x^3 - y^3 - z^3 \\
=& (x+y)^3 + z^3 + 3(x+y)z(x+y+z) - x^3 - y^3 - z^3 \\
=& (x+y)^3 + 3(x+y)z(x+y+z) - x^3 - y^3
\end{aligned}
$$

$$
\begin{aligned}
& (x+y+z)^3 - x^3 - y^3 - z^3 \\
=& \{(x+y)+z\}^3 - x^3 - y^3 - z^3 \\
=& (x+y)^3 + z^3 + 3(x+y)z\{(x+y)+z\} - x^3 - y^3 - z^3 \\
=& (x+y)^3 + z^3 + 3(x+y)z(x+y+z) - x^3 - y^3 - z^3 \\
=& (x+y)^3 + 3(x+y)z(x+y+z) - x^3 - y^3 \\
=& x^3 + y^3 + 3xy(x+y) + 3(x+y)z(x+y+z) - x^3 - y^3
\end{aligned}
$$

$$
\begin{aligned}
& (x+y+z)^3 - x^3 - y^3 - z^3 \\
=& \{(x+y)+z\}^3 - x^3 - y^3 - z^3 \\
=& (x+y)^3 + z^3 + 3(x+y)z\{(x+y)+z\} - x^3 - y^3 - z^3 \\
=& (x+y)^3 + z^3 + 3(x+y)z(x+y+z) - x^3 - y^3 - z^3 \\
=& (x+y)^3 + 3(x+y)z(x+y+z) - x^3 - y^3 \\
=& x^3 + y^3 + 3xy(x+y) + 3(x+y)z(x+y+z) - x^3 - y^3 \\
=& 3xy(x+y) + 3(x+y)z(x+y+z)
\end{aligned}
$$

$$
\begin{aligned}
& (x+y+z)^3 - x^3 - y^3 - z^3 \\
=& \{(x+y)+z\}^3 - x^3 - y^3 - z^3 \\
=& (x+y)^3 + z^3 + 3(x+y)z\{(x+y)+z\} - x^3 - y^3 - z^3 \\
=& (x+y)^3 + z^3 + 3(x+y)z(x+y+z) - x^3 - y^3 - z^3 \\
=& (x+y)^3 + 3(x+y)z(x+y+z) - x^3 - y^3 \\
=& x^3 + y^3 + 3xy(x+y) + 3(x+y)z(x+y+z) - x^3 - y^3 \\
=& 3xy(x+y) + 3(x+y)z(x+y+z) \\
=& 3(x+y)\{xy + z(x+y+z)\}
\end{aligned}
$$

$$
\begin{aligned}
& (x+y+z)^3 - x^3 - y^3 - z^3 \\
=& \{(x+y)+z\}^3 - x^3 - y^3 - z^3 \\
=& (x+y)^3 + z^3 + 3(x+y)z\{(x+y)+z\} - x^3 - y^3 - z^3 \\
=& (x+y)^3 + z^3 + 3(x+y)z(x+y+z) - x^3 - y^3 - z^3 \\
=& (x+y)^3 + 3(x+y)z(x+y+z) - x^3 - y^3 \\
=& x^3 + y^3 + 3xy(x+y) + 3(x+y)z(x+y+z) - x^3 - y^3 \\
=& 3xy(x+y) + 3(x+y)z(x+y+z) \\
=& 3(x+y)\{xy + z(x+y+z)\} \\
=& 3(x+y)(xy+zx+zy+z^2)
\end{aligned}
$$

$$
\begin{aligned}
& (x+y+z)^3 - x^3 - y^3 - z^3 \\
=& \{(x+y)+z\}^3 - x^3 - y^3 - z^3 \\
=& (x+y)^3 + z^3 + 3(x+y)z\{(x+y)+z\} - x^3 - y^3 - z^3 \\
=& (x+y)^3 + z^3 + 3(x+y)z(x+y+z) - x^3 - y^3 - z^3 \\
=& (x+y)^3 + 3(x+y)z(x+y+z) - x^3 - y^3 \\
=& x^3 + y^3 + 3xy(x+y) + 3(x+y)z(x+y+z) - x^3 - y^3 \\
=& 3xy(x+y) + 3(x+y)z(x+y+z) \\
=& 3(x+y)\{xy + z(x+y+z)\} \\
=& 3(x+y)(xy + zx + zy + z^2) \\
=& 3(x+y)\{z^2 + (x+y)z + xy\}
\end{aligned}
$$

$$
\begin{aligned}
& (x+y+z)^3 - x^3 - y^3 - z^3 \\
=& \{(x+y)+z\}^3 - x^3 - y^3 - z^3 \\
=& (x+y)^3 + z^3 + 3(x+y)z\{(x+y)+z\} - x^3 - y^3 - z^3 \\
=& (x+y)^3 + z^3 + 3(x+y)z(x+y+z) - x^3 - y^3 - z^3 \\
=& (x+y)^3 + 3(x+y)z(x+y+z) - x^3 - y^3 \\
=& x^3 + y^3 + 3xy(x+y) + 3(x+y)z(x+y+z) - x^3 - y^3 \\
=& 3xy(x+y) + 3(x+y)z(x+y+z) \\
=& 3(x+y)\{xy + z(x+y+z)\} \\
=& 3(x+y)(xy + zx + zy + z^2) \\
=& 3(x+y)\{z^2 + (x+y)z + xy\} \\
=& 3(x+y)(z+x)(z+y)
\end{aligned}
$$

$$
\begin{aligned}
& (x+y+z)^3 - x^3 - y^3 - z^3 \\
=& \{(x+y)+z\}^3 - x^3 - y^3 - z^3 \\
=& (x+y)^3 + z^3 + 3(x+y)z\{(x+y)+z\} - x^3 - y^3 - z^3 \\
=& (x+y)^3 + z^3 + 3(x+y)z(x+y+z) - x^3 - y^3 - z^3 \\
=& (x+y)^3 + 3(x+y)z(x+y+z) - x^3 - y^3 \\
=& x^3 + y^3 + 3xy(x+y) + 3(x+y)z(x+y+z) - x^3 - y^3 \\
=& 3xy(x+y) + 3(x+y)z(x+y+z) \\
=& 3(x+y)\{xy + z(x+y+z)\} \\
=& 3(x+y)(xy + zx + zy + z^2) \\
=& 3(x+y)\{z^2 + (x+y)z + xy\} \\
=& 3(x+y)(z+x)(z+y) \\
=& 3(x+y)(y+z)(z+x)
\end{aligned}
$$

Github:
https://min7014.github.io/math20200326002.html

Click or paste URL into the URL search bar, and you can see a picture moving.