함수의 합성
(Function Composition)

Start End

Start End

Function composition

Start End

Function composition is

Start End

Function composition is the pointwise application

Start End

Function composition is the pointwise application of

Start End

Function composition is the pointwise application of one function

Start End

Function composition is the pointwise application of one function to the result of

Start End

Function composition is the pointwise application of one function to the result of another

Start End

Function composition is the pointwise application of one function to the result of another to

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function.

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance,

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : \mathrm{X} \to \mathrm{Y}$

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : \mathrm{X} \to \mathrm{Y}$ and

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : \mathrm{X} \to \mathrm{Y}$ and $g : \mathrm{Y} \to \mathrm{Z}$ can be composed

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : \mathrm{X} \to \mathrm{Y}$ and $g : \mathrm{Y} \to \mathrm{Z}$ can be composed to yield a function

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : \mathrm{X} \to \mathrm{Y}$ and $g : \mathrm{Y} \to \mathrm{Z}$ can be composed to yield a function which maps

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : \mathrm{X} \to \mathrm{Y}$ and $g : \mathrm{Y} \to \mathrm{Z}$ can be composed to yield a function which maps x

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : \mathrm{X} \to \mathrm{Y}$ and $g : \mathrm{Y} \to \mathrm{Z}$ can be composed to yield a function which maps x in X

▸ Start ▸ End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : \mathrm{X} \to \mathrm{Y}$ and $g : \mathrm{Y} \to \mathrm{Z}$ can be composed to yield a function which maps x in X to $g(f(x))$

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : \mathrm{X} \to \mathrm{Y}$ and $g : \mathrm{Y} \to \mathrm{Z}$ can be composed to yield a function which maps x in X to $g(f(x))$ in

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : \mathrm{X} \to \mathrm{Y}$ and $g : \mathrm{Y} \to \mathrm{Z}$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z.

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively,

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : \mathrm{X} \to \mathrm{Y}$ and $g : \mathrm{Y} \to \mathrm{Z}$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y,

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : \mathrm{X} \to \mathrm{Y}$ and $g : \mathrm{Y} \to \mathrm{Z}$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and

▸ Start ▸ End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : \mathrm{X} \to \mathrm{Y}$ and $g : \mathrm{Y} \to \mathrm{Z}$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : \mathrm{X} \to \mathrm{Y}$ and $g : \mathrm{Y} \to \mathrm{Z}$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x,

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : \mathrm{X} \to \mathrm{Y}$ and $g : \mathrm{Y} \to \mathrm{Z}$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : \mathrm{X} \to \mathrm{Y}$ and $g : \mathrm{Y} \to \mathrm{Z}$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : \mathrm{X} \to \mathrm{Y}$ and $g : \mathrm{Y} \to \mathrm{Z}$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : \mathrm{X} \to \mathrm{Y}$ and $g : \mathrm{Y} \to \mathrm{Z}$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x.

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : \mathrm{X} \to \mathrm{Y}$ and $g : \mathrm{Y} \to \mathrm{Z}$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : \mathrm{X} \to \mathrm{Y}$ and $g : \mathrm{Y} \to \mathrm{Z}$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : \mathrm{X} \to \mathrm{Y}$ and $g : \mathrm{Y} \to \mathrm{Z}$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : \mathrm{X} \to \mathrm{Z}$,

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : \mathrm{X} \to \mathrm{Y}$ and $g : \mathrm{Y} \to \mathrm{Z}$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : \mathrm{X} \to \mathrm{Z}$, defined by $(g \circ f)(x) = g(f(x))$

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : \mathrm{X} \to \mathrm{Y}$ and $g : \mathrm{Y} \to \mathrm{Z}$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : \mathrm{X} \to \mathrm{Z}$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : \mathrm{X} \to \mathrm{Y}$ and $g : \mathrm{Y} \to \mathrm{Z}$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : \mathrm{X} \to \mathrm{Z}$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f",

▸ Start ▸ End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.

The notation $g \circ f$ is read as "g circle f", or "g round f",

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f",

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f",

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f",

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x.
The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f".

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative.

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is,

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if f

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : \mathrm{X} \to \mathrm{Y}$ and $g : \mathrm{Y} \to \mathrm{Z}$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : \mathrm{X} \to \mathrm{Z}$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if f, g

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if f, g, and h

▸ Start ▸ End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if f, g, and h are

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.

The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if f, g, and h are three functions

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if f, g, and h are three functions with suitably chosen

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : \mathrm{X} \to \mathrm{Y}$ and $g : \mathrm{Y} \to \mathrm{Z}$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : \mathrm{X} \to \mathrm{Z}$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.

The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if f, g, and h are three functions with suitably chosen domains

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if f, g, and h are three functions with suitably chosen domains and codomains,

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : \mathrm{X} \to \mathrm{Y}$ and $g : \mathrm{Y} \to \mathrm{Z}$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : \mathrm{X} \to \mathrm{Z}$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if f, g, and h are three functions with suitably chosen domains and codomains, then $f \circ (g \circ h) = (f \circ g) \circ h$,

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if f, g, and h are three functions with suitably chosen domains and codomains, then $f \circ (g \circ h) = (f \circ g) \circ h$, where

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if f, g, and h are three functions with suitably chosen domains and codomains, then $f \circ (g \circ h) = (f \circ g) \circ h$, where the parentheses serve

▸ Start ▸ End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if f, g, and h are three functions with suitably chosen domains and codomains, then $f \circ (g \circ h) = (f \circ g) \circ h$, where the parentheses serve to indicate

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if f, g, and h are three functions with suitably chosen domains and codomains, then $f \circ (g \circ h) = (f \circ g) \circ h$, where the parentheses serve to indicate that composition

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if f, g, and h are three functions with suitably chosen domains and codomains, then $f \circ (g \circ h) = (f \circ g) \circ h$, where the parentheses serve to indicate that composition is

▸ Start ▸ End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.

The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if f, g, and h are three functions with suitably chosen domains and codomains, then $f \circ (g \circ h) = (f \circ g) \circ h$, where the parentheses serve to indicate that composition is to be performed

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.

The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if f, g, and h are three functions with suitably chosen domains and codomains, then $f \circ (g \circ h) = (f \circ g) \circ h$, where the parentheses serve to indicate that composition is to be performed first

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if f, g, and h are three functions with suitably chosen domains and codomains, then $f \circ (g \circ h) = (f \circ g) \circ h$, where the parentheses serve to indicate that composition is to be performed first for the parenthesized functions.

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if f, g, and h are three functions with suitably chosen domains and codomains, then $f \circ (g \circ h) = (f \circ g) \circ h$, where the parentheses serve to indicate that composition is to be performed first for the parenthesized functions. Since

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if f, g, and h are three functions with suitably chosen domains and codomains, then $f \circ (g \circ h) = (f \circ g) \circ h$, where the parentheses serve to indicate that composition is to be performed first for the parenthesized functions. Since there is

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if f, g, and h are three functions with suitably chosen domains and codomains, then $f \circ (g \circ h) = (f \circ g) \circ h$, where the parentheses serve to indicate that composition is to be performed first for the parenthesized functions. Since there is no distinction

▸ Start ▸ End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if f, g, and h are three functions with suitably chosen domains and codomains, then $f \circ (g \circ h) = (f \circ g) \circ h$, where the parentheses serve to indicate that composition is to be performed first for the parenthesized functions. Since there is no distinction between the choices of

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if f, g, and h are three functions with suitably chosen domains and codomains, then $f \circ (g \circ h) = (f \circ g) \circ h$, where the parentheses serve to indicate that composition is to be performed first for the parenthesized functions. Since there is no distinction between the choices of placement of

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if f, g, and h are three functions with suitably chosen domains and codomains, then $f \circ (g \circ h) = (f \circ g) \circ h$, where the parentheses serve to indicate that composition is to be performed first for the parenthesized functions. Since there is no distinction between the choices of placement of parentheses,

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if f, g, and h are three functions with suitably chosen domains and codomains, then $f \circ (g \circ h) = (f \circ g) \circ h$, where the parentheses serve to indicate that composition is to be performed first for the parenthesized functions. Since there is no distinction between the choices of placement of parentheses, they

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if f, g, and h are three functions with suitably chosen domains and codomains, then $f \circ (g \circ h) = (f \circ g) \circ h$, where the parentheses serve to indicate that composition is to be performed first for the parenthesized functions. Since there is no distinction between the choices of placement of parentheses, they may be left off

▸ Start ▸ End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if f, g, and h are three functions with suitably chosen domains and codomains, then $f \circ (g \circ h) = (f \circ g) \circ h$, where the parentheses serve to indicate that composition is to be performed first for the parenthesized functions. Since there is no distinction between the choices of placement of parentheses, they may be left off without causing

Start End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : X \to Y$ and $g : Y \to Z$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : X \to Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.
The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if f, g, and h are three functions with suitably chosen domains and codomains, then $f \circ (g \circ h) = (f \circ g) \circ h$, where the parentheses serve to indicate that composition is to be performed first for the parenthesized functions. Since there is no distinction between the choices of placement of parentheses, they may be left off without causing any ambiguity.

▸ Start ▸ End

Function composition is the pointwise application of one function to the result of another to produce a third function. For instance, the functions $f : \mathrm{X} \to \mathrm{Y}$ and $g : \mathrm{Y} \to \mathrm{Z}$ can be composed to yield a function which maps x in X to $g(f(x))$ in Z. Intuitively, if z is a function g of y, and y is a function f of x, then z is a function of x. The resulting composite function is notated $g \circ f : \mathrm{X} \to \mathrm{Z}$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X.

The notation $g \circ f$ is read as "g circle f", or "g round f", or "g composed with f", or "g after f", or "g following f", or "g of f". The composition of functions is always associative. That is, if f, g, and h are three functions with suitably chosen domains and codomains, then $f \circ (g \circ h) = (f \circ g) \circ h$, where the parentheses serve to indicate that composition is to be performed first for the parenthesized functions. Since there is no distinction between the choices of placement of parentheses, they may be left off without causing any ambiguity.

X

$$X \xrightarrow{f}$$

$$X \xrightarrow{f} \quad Y$$

Home Start End

$$X \xrightarrow{f} Y \xrightarrow{g}$$

Home Start End

$$X \xrightarrow{f} Y \xrightarrow{g} Z$$

Home Start End

$$X \xrightarrow{f} Y \xrightarrow{g} Z$$

$$x$$

Home Start End

$$
\begin{array}{ccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\
x & \xrightarrow{f} & & &
\end{array}
$$

Home Start End

$$
\begin{array}{ccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\
x & \xrightarrow{f} & y & &
\end{array}
$$

$$\begin{array}{ccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & \end{array}$$

$$
\begin{array}{ccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z
\end{array}
$$

Home Start End

$$X \xrightarrow{f} Y \xrightarrow{g} Z$$

$$x \xrightarrow{f} y \xrightarrow{g} z$$

$$x$$

Home Start End

$$
\begin{array}{ccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z \\
x & \xrightarrow{f} & & &
\end{array}
$$

Home Start End

$$
\begin{array}{ccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z \\
x & \xrightarrow{f} & f(x) & &
\end{array}
$$

Home Start End

$$
\begin{array}{ccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} &
\end{array}
$$

$$\begin{array}{ccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) \end{array}$$

$$
\begin{array}{ccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & &
\end{array}
$$

Home Start End

$$X \xrightarrow{f} Y \xrightarrow{g} Z \qquad X \xrightarrow{g\circ f}$$

$$x \xrightarrow{f} y \xrightarrow{g} z$$

$$x \xrightarrow{f} f(x) \xrightarrow{g} g(f(x))$$

Home Start End

$$X \xrightarrow{f} Y \xrightarrow{g} Z \qquad X \xrightarrow{g \circ f} Z$$

$$x \xrightarrow{f} y \xrightarrow{g} z$$

$$x \xrightarrow{f} f(x) \xrightarrow{g} g(f(x))$$

Home Start End

$$
\begin{array}{ccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X \xrightarrow{g\circ f} Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & \quad & x \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & &
\end{array}
$$

$$\begin{array}{ccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) \end{array}$$

$$\begin{array}{ccc} X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{g \circ f} & \end{array}$$

Home Start End

$$X \xrightarrow{f} Y \xrightarrow{g} Z \qquad X \xrightarrow{g\circ f} Z$$

$$x \xrightarrow{f} y \xrightarrow{g} z \qquad x \xrightarrow{g\circ f} z$$

$$x \xrightarrow{f} f(x) \xrightarrow{g} g(f(x))$$

Home Start End

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & &
\end{array}
$$

Home Start End

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & \quad & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & \quad & x & \xrightarrow{g\circ f} & \end{array}$$

Home Start End

$$
\begin{array}{ccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x))
\end{array}
\qquad
\begin{array}{ccc}
X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$
\begin{array}{ccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x))
\end{array}
\qquad
\begin{array}{ccc}
\mathrm{X} & \xrightarrow{g \circ f} & \mathrm{Z} \\
x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore$$

Home Start End

$$
\begin{array}{ccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x))
\end{array}
\qquad
\begin{array}{ccc}
X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x$$

▸ Home ▸ Start ▸ End

$$\begin{array}{ccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) \end{array} \qquad \begin{array}{ccc} X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore \; \forall x \in X,$$

Home Start End

$$
\begin{array}{ccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x))
\end{array}
\qquad
\begin{array}{ccc}
X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ ($$

Home Start End

$$\begin{array}{ccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) \end{array} \qquad \begin{array}{ccc} X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g$$

$$\begin{array}{ccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) \end{array} \qquad \begin{array}{ccc} X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g \circ$$

Home Start End

$$
\begin{array}{ccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x))
\end{array}
\qquad
\begin{array}{ccc}
X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f$$

$$X \xrightarrow{f} Y \xrightarrow{g} Z \qquad X \xrightarrow{g\circ f} Z$$

$$x \xrightarrow{f} y \xrightarrow{g} z \qquad x \xrightarrow{g\circ f} z$$

$$x \xrightarrow{f} f(x) \xrightarrow{g} g(f(x)) \qquad x \xrightarrow{g\circ f} (g\circ f)(x)$$

$$\therefore\ \forall x \in X,\ (g \circ f)$$

Home Start End

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x)$$

▸ Home ▸ Start ▸ End

$$\begin{array}{ccccccccc} \mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & \quad & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in \mathrm{X},\ (g\circ f)(x) =$$

Home Start End

$$
\begin{array}{ccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & \qquad x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & \qquad x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g$$

Home Start End

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g($$

Home Start End

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f$$

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

Home Start End

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$
\begin{array}{ccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

(

$$\begin{array}{ccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$((($

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$((h$

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$((h \circ$

▸ Home ▸ Start ▸ End

$$
\begin{array}{ccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & \qquad x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & \qquad x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h \circ g$$

▸ Home ▸ Start ▸ End

$$
\begin{array}{ccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & \qquad x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & \qquad x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$(\,(h\circ g)$$

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$(\,(h\circ g)\ \circ$$

Home Start End

$$
\begin{array}{ccccccccc}
\mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in \mathrm{X},\ (g\circ f)(x) = g(f(x))$$

$$(\,(h\circ g)\ \circ f$$

Home Start End

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore \ \forall x \in X, \ (g \circ f)(x) = g(f(x))$$

$$(\,(h \circ g)\ \circ f)$$

Home Start End

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x)$$

Home Start End

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$\left((h \circ g)\ \circ f \right) (x) =$$

Home Start End

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = ($$

▸ Home ▸ Start ▸ End

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$\left(\left(h \circ g\right) \circ f\right)(x) = \left(h\right.$$

Home Start End

$$
\begin{array}{ccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ$$

Home Start End

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g$$

Home Start End

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)$$

▸ Home ▸ Start ▸ End

$$\begin{array}{ccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & \qquad x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & \qquad x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)\ ($$

Home Start End

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)\ (f$$

Home Start End

$$
\begin{array}{ccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & \quad x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & \quad x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore \ \forall x \in X, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x))$$

▸ Home ▸ Start ▸ End

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore \ \forall x \in X, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x))$$

Home Start End

$$\begin{array}{ccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h$$

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h($$

▸ Home ▸ Start ▸ End

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g$$

▸ Home ▸ Start ▸ End

$$X \xrightarrow{f} Y \xrightarrow{g} Z \qquad X \xrightarrow{g \circ f} Z$$

$$x \xrightarrow{f} y \xrightarrow{g} z \qquad x \xrightarrow{g \circ f} z$$

$$x \xrightarrow{f} f(x) \xrightarrow{g} g(f(x)) \qquad x \xrightarrow{g \circ f} (g \circ f)(x)$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g($$

$$
\begin{array}{ccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f$$

▸ Home ▸ Start ▸ End

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)$$

▸ Home ▸ Start ▸ End

$$
\begin{array}{ccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x))
\end{array}
\qquad
\begin{array}{ccc}
X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

▸ Home ▸ Start ▸ End

$$
\begin{array}{ccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

▸ Home ▸ Start ▸ End

$$X \xrightarrow{f} Y \xrightarrow{g} Z \qquad X \xrightarrow{g\circ f} Z$$
$$x \xrightarrow{f} y \xrightarrow{g} z \qquad x \xrightarrow{g\circ f} z$$
$$x \xrightarrow{f} f(x) \xrightarrow{g} g(f(x)) \qquad x \xrightarrow{g\circ f} (g\circ f)(x)$$
$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$=$$

Home Start End

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h$$

$$
\begin{array}{ccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x))
\end{array}
\qquad
\begin{array}{ccc}
X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h($$

▸ Home ▸ Start ▸ End

$$\begin{array}{ccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) \end{array} \qquad \begin{array}{ccc} X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$\left(\left(h\circ g\right)\circ f\right)\left(x\right) = \left(h\circ g\right)\left(f\left(x\right)\right) = h\left(g\left(f\left(x\right)\right)\right)$$

$$= h\left(\left(\right.\right.$$

Home Start End

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g$$

▸ Home ▸ Start ▸ End

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ$$

▸ Home ▸ Start ▸ End

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore \ \forall x \in X, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f$$

▸ Home ▸ Start ▸ End

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)$$

Home Start End

$$X \xrightarrow{f} Y \xrightarrow{g} Z \qquad X \xrightarrow{g\circ f} Z$$
$$x \xrightarrow{f} y \xrightarrow{g} z \qquad x \xrightarrow{g\circ f} z$$
$$x \xrightarrow{f} f(x) \xrightarrow{g} g(f(x)) \qquad x \xrightarrow{g\circ f} (g\circ f)(x)$$
$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x))$$

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore \ \forall x \in X, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x))$$

▸ Home ▸ Start ▸ End

$$\begin{array}{ccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) \end{array} \qquad \begin{array}{ccc} X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) =$$

Home Start End

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = ($$

Home Start End

$$
\begin{array}{ccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h$$

Home Start End

$$
\begin{array}{ccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x))
\end{array}
\qquad
\begin{array}{ccc}
X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore \ \forall x \in X, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ$$

Home Start End

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ ($$

Home Start End

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g$$

▸ Home ▸ Start ▸ End

$$\begin{array}{ccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) \end{array} \qquad \begin{array}{ccc} X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ$$

Home Start End

$$\begin{array}{ccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) \end{array} \qquad \begin{array}{ccc} X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f$$

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & \qquad & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & \qquad & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)\ (f\ (x)\ ) = h\,(g\,(f\ (x)\ )\ )$$

$$= h(\ (g\circ f)\ (x)\ ) = (h\circ\ (g\circ f)$$

▸ Home ▸ Start ▸ End

$$\begin{array}{ccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & \quad x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & \quad x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore \ \forall x \in X, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))$$

Home Start End

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))($$

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ(g\circ f))(x)$$

Home Start End

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & \qquad & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & \qquad & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore \ \forall x \in X, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore$$

▸ Home ▸ Start ▸ End

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x$$

Home Start End

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x \in X, ($$

▸ Home ▸ Start ▸ End

$$\begin{array}{ccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) \end{array} \qquad \begin{array}{ccc} X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore \ \forall x \in X, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore \ \forall x \in X, ((\ $$

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore \ \forall x \in X, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore \ \forall x \in X, ((h$$

▸ Home ▸ Start ▸ End

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore \ \forall x \in X, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore \ \forall x \in X, ((h \circ$$

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore \ \forall x \in X, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore \ \forall x \in X, ((h \circ g$$

$$\begin{array}{ccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x \in X, ((h \circ g)$$

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ(g\circ f))(x)$$

$$\therefore\ \forall x \in X, ((h\circ g)\circ$$

▸ Home ▸ Start ▸ End

$$
\begin{array}{ccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x))
\end{array}
\qquad
\begin{array}{ccc}
X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x \in X, ((h \circ g) \circ f$$

▸ Home ▸ Start ▸ End

$$\begin{array}{ccccccccc} \mathrm{X} & \xrightarrow{f} & \mathrm{Y} & \xrightarrow{g} & \mathrm{Z} & \quad & \mathrm{X} & \xrightarrow{g\circ f} & \mathrm{Z} \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in \mathrm{X},\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ (g\circ f))(x)$$

$$\therefore\ \forall x \in \mathrm{X}, ((h\circ g)\circ f)$$

▸ Home ▸ Start ▸ End

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ (g\circ f))(x)$$

$$\therefore\ \forall x \in X, ((h\circ g)\circ f)(x)$$

$$X \xrightarrow{f} Y \xrightarrow{g} Z \qquad X \xrightarrow{g \circ f} Z$$
$$x \xrightarrow{f} y \xrightarrow{g} z \qquad x \xrightarrow{g \circ f} z$$
$$x \xrightarrow{f} f(x) \xrightarrow{g} g(f(x)) \qquad x \xrightarrow{g \circ f} (g \circ f)(x)$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x \in X, ((h \circ g) \circ f)(x) =$$

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x \in X, ((h \circ g) \circ f)(x) = ($$

▸ Home ▸ Start ▸ End

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ(g\circ f))(x)$$

$$\therefore\ \forall x \in X, ((h\circ g)\circ f)(x) = (h$$

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ (g\circ f))(x)$$

$$\therefore\ \forall x \in X, ((h\circ g)\circ f)(x) = (h\circ$$

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ (g\circ f))(x)$$

$$\therefore\ \forall x \in X, ((h\circ g)\circ f)(x) = (h\circ ($$

Home Start End

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore \ \forall x \in X, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore \ \forall x \in X, ((h \circ g) \circ f)(x) = (h \circ (g$$

Home Start End

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ (g\circ f))(x)$$

$$\therefore\ \forall x \in X, ((h\circ g)\circ f)(x) = (h\circ (g\circ$$

▸ Home ▸ Start ▸ End

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x \in X, ((h \circ g) \circ f)(x) = (h \circ (g \circ f$$

Home Start End

$$
\begin{array}{ccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x))
\end{array}
\qquad
\begin{array}{ccc}
X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore \ \forall x \in X, \ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ (g\circ f))(x)$$

$$\therefore \ \forall x \in X, ((h\circ g)\circ f)(x) = (h\circ (g\circ f)$$

Home Start End

$$\begin{array}{ccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x))
\end{array}
\qquad
\begin{array}{ccc}
X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x \in X, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))$$

$$X \xrightarrow{f} Y \xrightarrow{g} Z \qquad X \xrightarrow{g \circ f} Z$$

$$x \xrightarrow{f} y \xrightarrow{g} z \qquad x \xrightarrow{g \circ f} z$$

$$x \xrightarrow{f} f(x) \xrightarrow{g} g(f(x)) \qquad x \xrightarrow{g \circ f} (g \circ f)(x)$$

$$\therefore \ \forall x \in X, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore \ \forall x \in X, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

Home Start End

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x \in X, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

(

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ (g\circ f))(x)$$

$$\therefore\ \forall x \in X, ((h\circ g)\circ f)(x) = (h\circ (g\circ f))(x)$$

$$(h$$

Home Start End

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ (g\circ f))(x)$$

$$\therefore\ \forall x \in X, ((h\circ g)\circ f)(x) = (h\circ (g\circ f))(x)$$

$$(h\circ$$

Home Start End

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x \in X, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ g$$

Home Start End

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x \in X, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ g)$$

▸ Home ▸ Start ▸ End

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore \ \forall x \in X, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore \ \forall x \in X, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ g) \circ$$

Home Start End

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x \in X, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ g) \circ f$$

▸ Home ▸ Start ▸ End

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad & X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x \in X, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ g) \circ f =$$

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x \in X, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ g) \circ f = h$$

$$
\begin{array}{ccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \qquad X & \xrightarrow{g\circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & \qquad x & \xrightarrow{g\circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & \qquad x & \xrightarrow{g\circ f} & (g\circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ(g\circ f))(x)$$

$$\therefore\ \forall x \in X, ((h\circ g)\circ f)(x) = (h\circ(g\circ f))(x)$$

$$(h\circ g)\circ f = h\circ$$

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore \ \forall x \in X, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore \ \forall x \in X, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ g) \circ f = h \circ ($$

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g \circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x) \end{array}$$

$$\therefore \ \forall x \in X, \ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore \ \forall x \in X, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ g) \circ f = h \circ (g$$

▸ Home ▸ Start ▸ End

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x \in X, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ g) \circ f = h \circ (g \circ$$

Home Start End

$$
\begin{array}{ccccccccc}
X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & & X & \xrightarrow{g \circ f} & Z \\
x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g \circ f} & z \\
x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g \circ f} & (g \circ f)(x)
\end{array}
$$

$$\therefore\ \forall x \in X,\ (g \circ f)(x) = g(f(x))$$

$$((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$$

$$= h((g \circ f)(x)) = (h \circ (g \circ f))(x)$$

$$\therefore\ \forall x \in X, ((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$$

$$(h \circ g) \circ f = h \circ (g \circ f$$

▸ Home ▸ Start ▸ End

$$\begin{array}{ccccccccc} X & \xrightarrow{f} & Y & \xrightarrow{g} & Z & \quad & X & \xrightarrow{g\circ f} & Z \\ x & \xrightarrow{f} & y & \xrightarrow{g} & z & & x & \xrightarrow{g\circ f} & z \\ x & \xrightarrow{f} & f(x) & \xrightarrow{g} & g(f(x)) & & x & \xrightarrow{g\circ f} & (g\circ f)(x) \end{array}$$

$$\therefore\ \forall x \in X,\ (g\circ f)(x) = g(f(x))$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = h(g(f(x)))$$

$$= h((g\circ f)(x)) = (h\circ (g\circ f))(x)$$

$$\therefore\ \forall x \in X, ((h\circ g)\circ f)(x) = (h\circ (g\circ f))(x)$$

$$(h\circ g)\circ f = h\circ (g\circ f)$$

Github:
https://min7014.github.io/math20190927003.html

Click or paste URL into the URL search bar, and you can see a picture moving.