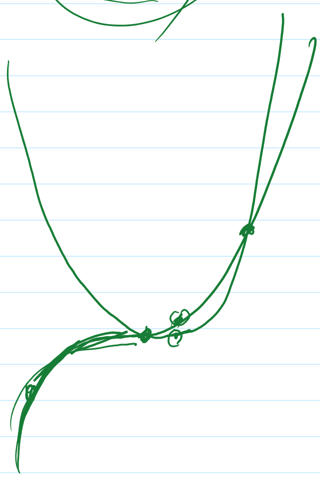
* $f(x) = \begin{cases} 1, & x = 0 \\ \dfrac{\sin x}{x}, & x \neq 0 \end{cases}$

f가 $\mathbb{R}$에서 연속임을 보이시오.

* $\begin{cases} f: \mathbb{R} \to \mathbb{R}, \quad g: \mathbb{R} \to \mathbb{R} \\ \forall x \in \mathbb{R}, \ f(x) \cdot g(x) = x^5 \quad f(x) + g(x) = x^2 + x^3 \end{cases}$

i) 이것을 만족하는 함수 f, g를 모두 구하시오

ii) f, g가 $\mathbb{R}$에서 연속일 때 만족하는 f, g를 구하시오.



* $\begin{cases} f: \mathbb{R} \to \mathbb{R}, \quad g: \mathbb{R} \to \mathbb{R} \\ \forall x \in \mathbb{R}, \ f(x) \cdot g(x) = h(x) \cdot k(x) \quad f(x) + g(x) = h(x) + k(x) \end{cases}$ (h, k는 연속)

$h(x) = k(x)$의 서로 다른 해가 n개라면

f, g가 $\mathbb{R}$에서 연속인 가능한 순서쌍 개수 $\boxed{2^{n+1}}$

* $\begin{cases} f: \mathbb{R} \to \mathbb{R} \end{cases}$

* $\begin{cases} f : \mathbb{R} \to \mathbb{R} \\ \text{모든 } x \in \mathbb{R} \text{ 에서 함수 } f \text{ 가 불연속이다.} \end{cases}$

위 조건을 만족하는 함수 f를 예로 들어 보세요.

ex.) $f(x) = \begin{cases} 1 & , x \in \mathbb{Q} \\ 0 & , x \in \mathbb{Q}^c \end{cases}$ ← 동그라미+ㅅ

* $\begin{cases} f : \mathbb{R} \to \mathbb{R} \\ f \text{가 } 0 \text{에서만 연속이다.} \end{cases}$

위 조건을 만족하는 함수 f를 예로 들어 보세요

ex) $g(x) = \begin{cases} x & , x \in \mathbb{Q} \\ 0 & , x \in \mathbb{Q}^c \end{cases}$ $\underline{g(x) = x f(x)}$

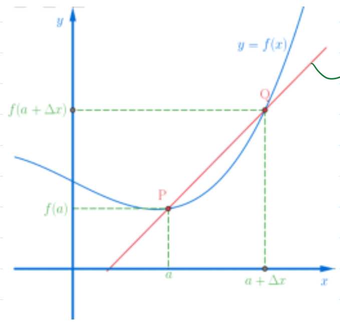
추가 질문 n개에서 연속인 함수는?

$h(x) = (x-1)(x-2)\cdots(x-n) \cdot f(x)$

미분계수

2019년 7월 22일 월요일 오전 8:42

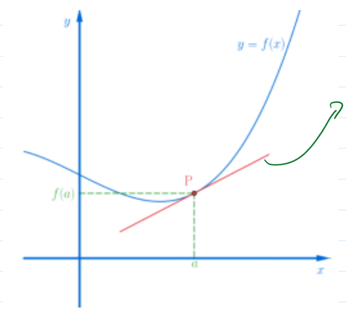
$$\left[\frac{\Delta y}{\Delta x}\right]_{P.Q,\ y=f(x)} = \boxed{\left[\frac{\Delta y}{\Delta x}\right]_{x=a,\ \Delta x,\ y=f(x)}}$$



$y = mx + n$

└> 평균변화율

$$\left[\frac{\Delta y}{\Delta x}\right]_{P.Q,\ y=f(x)} = \left[\frac{\Delta y}{\Delta x}\right]_{x=a,\Delta x,\ y=f(x)}$$



$$\left[\frac{dy}{dx}\right]_{x=a,\ y=f(x)} = \lim_{\Delta x \to 0}\left[\frac{\Delta y}{\Delta x}\right]_{\substack{x=a \\ \Delta x \\ y=f(x)}}$$

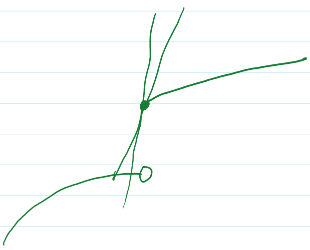
$$\|$$

$$\left[\frac{dy}{dx}\right]_{x=a} = y'_{x=a} = f'(a)$$

$$\left[\frac{\Delta y}{\Delta x}\right]_{\substack{x=a \\ y=f(x)}} \qquad \lim_{\Delta x \to 0}\left[\frac{\Delta y}{\Delta x}\right]_{\substack{x=a \\ y=f(x)}} = \left[\frac{dy}{dx}\right]_{x=a}$$

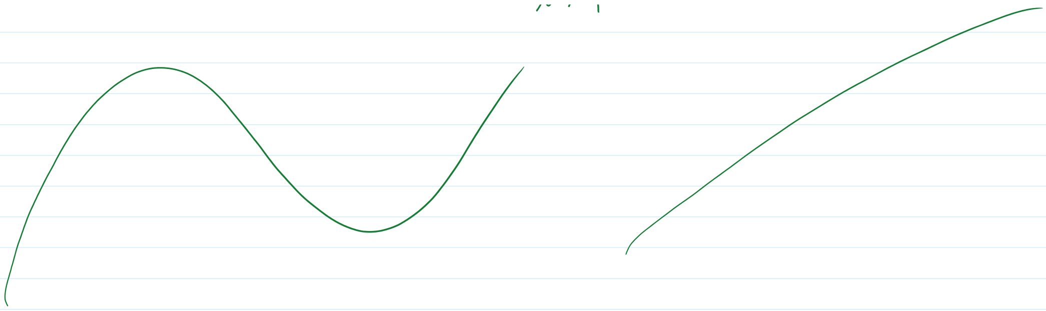
$$= f'(a)$$

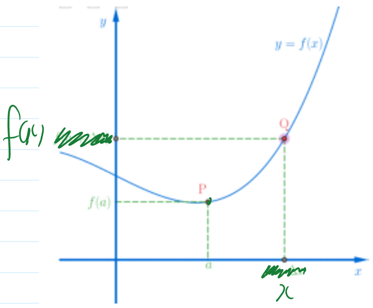
$$= y'_{x=a}$$



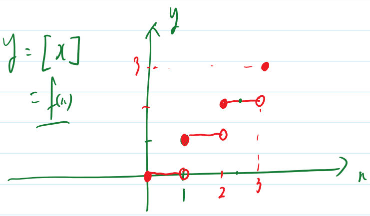
$\boxed{y = \sqrt{x}}$

$$\left(\frac{\sqrt{x} - 0}{x - 0} = \frac{1}{\sqrt{x}}\right. \qquad \lim_{x \to 0+}\frac{1}{\sqrt{x}} = \infty$$





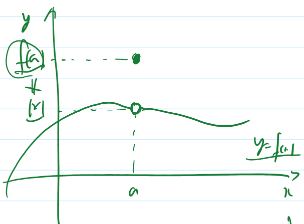
$$\boxed{\lim_{x\to a} f(x) = f(a)} \Leftarrow$$ 함수 f가 a에서 연속이다



$\lim_{x\to 2+} [x] = 2$

$\lim_{x\to 2-} [x] = 1$

$\nexists \lim_{x\to 2} [x]$



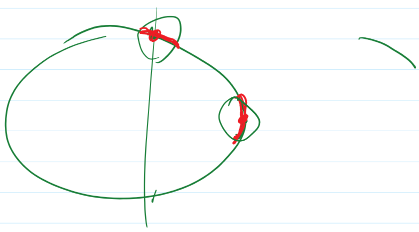
$\lim_{x\to a} f(x) = M \neq f(a)$

$f(x) = \begin{cases} x, & x\neq 0 \\ 1, & x=0 \end{cases}$ $\lim_{x\to 0} f(x) = 0 \neq f(0) = 1$

연속
- 극한값 존재
- 함숫값 존재
- 극한값 = 함숫값

$$\lim_{x\to a} \frac{f(x)-f(a)}{x-a} = f'(a)$$

$\forall \varepsilon > 0,\ \exists \delta > 0$ s.t. $0 < |x-a| < \delta \Rightarrow \left| \frac{f(x)-f(a)}{x-a} - f'(a) \right| < \varepsilon$



$f(x) - f(a)$

$$\lim_{x\to a}\{f(x)-f(a)\} = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}\cdot(x-a)$$

$$\lim_{x\to a}\{f(x)-f(a)\} = \lim_{x\to a}f(x) - \lim_{x\to a}f(a)$$

$$= \lim_{x\to a}f(x) - f(a)$$

$$= f'(a)\cdot 0 = 0$$

$$\therefore \lim_{x\to a}f(x) = f(a)$$

$$\lim_{x\to a}f(x) = L \qquad \forall\varepsilon>0,\ \exists\delta>0 \text{ s.t. } 0<|x-a|<\delta \Rightarrow |f(x)-L|<\varepsilon$$

$$\lim_{x\to a}\{f(x)-f(a)\}=0 \qquad \forall\varepsilon>0,\ \exists\delta>0 \text{ s.t. } 0<|x-a|<\delta \Rightarrow |\{f(x)-f(a)\}-0|<\varepsilon$$

$$|f(x)-f(a)|<\varepsilon$$

$$\lim_{x\to a}f(x) = f(a) \qquad \forall\varepsilon>0,\ \exists\delta>0 \text{ s.t. } 0<|x-a|<\delta \Rightarrow |f(x)-f(a)|<\varepsilon$$

$$\exists\lim_{x\to a}f(x),\ \exists\lim_{x\to a}g(x)$$

$$\Rightarrow \lim_{x\to a}f(x)\cdot g(x) = \left\{\lim_{x\to a}f(x)\right\}\cdot\left\{\lim_{x\to a}g(x)\right\}$$

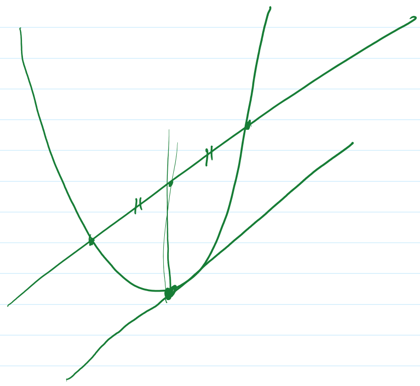
$$\lim_{x\to a}\{f(x)-g(x)\} = \lim_{x\to a}f(x) - \lim_{x\to a}g(x)$$

$$\lim_{x\to 0}1 = \lim_{x\to 0}x\cdot\frac{1}{x} = \lim_{x\to 0}x\cdot\lim_{x\to 0}\frac{1}{x}$$

$$\lim_{x\to 0}0 = \lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{x}\right) = \lim_{x\to 0}\frac{1}{x} - \lim_{x\to 0}\frac{1}{x}$$

$$\boxed{\lim_{x\to\infty}\frac{1}{x}=0} \qquad \boxed{\forall\varepsilon>0,\ \exists M \text{ s.t. } x>M \Rightarrow \left|\frac{1}{x}-0\right|<\varepsilon}$$

3-1-(3)



$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to a} \frac{f(a + x - a) - f(a)}{x - a}$$

$\Delta x = x - a$

$$= \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{\Delta x \to 0} \frac{f(a + \Delta x) - f(a)}{\Delta x}$$

$\lim_{x \to a} g(x) = L$ $\quad$ $\forall \varepsilon > 0, \exists \delta > 0$ s.t. $0 < |x - a| < \delta \Rightarrow |g(x) - L| < \varepsilon$

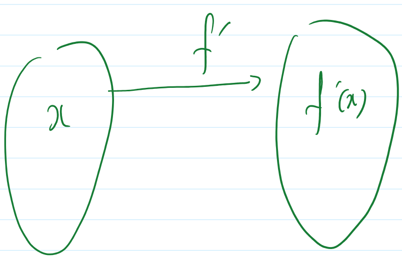
$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \qquad a \in [\alpha, \beta]$$

$a \Leftrightarrow x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad x \in [\alpha, \beta]$$

$\boxed{A \xrightarrow{f} B}$ $\quad$ $\forall x \in A, \exists y \in B$ s.t. $f(x) = y$

f를 함수라고 한다



$\{A \xrightarrow{f} B$

$$\begin{cases} A \xrightarrow{\;'\;} B \\ \quad f' \\ A' \longrightarrow B' \end{cases} \Rightarrow \quad A' \subset A$$

$*$ $f(x) = |x|$, $x \in \mathbb{R}$ $\quad f'(x)$?

답: $f'(x) = \begin{cases} -1 & , x < 0 \\ 1 & , x > 0 \end{cases}$

$*$ $f(x) = [x]$, $x \in \mathbb{R}$ $\quad f'(x)$?

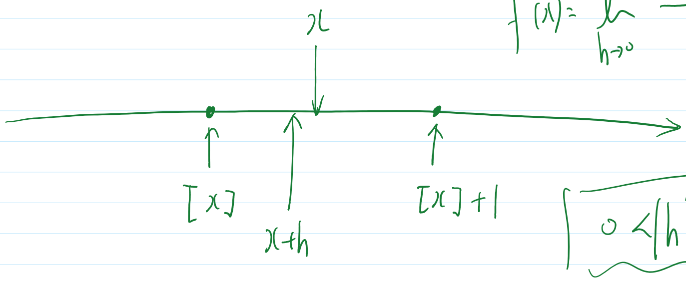
$x = n + a \quad 0 \le a < 1 ,\ n \in \mathbb{Z}$

답: $f'(x) = 0$, $\mathbb{R} - \mathbb{Z}$

$$\lim_{h \to 0} \frac{[x+h] - [x]}{h} = \lim_{h \to 0} \frac{[n+a+h] - [n+a]}{h} = \lim_{h \to 0} \frac{[n+a+h] - n}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{n} + [a+h] - \cancel{n}}{h} = \lim_{h \to 0} \frac{[a+h]}{h} = \begin{cases} \lim_{h \to 0} \frac{[h]}{h} & a = 0 \\ \lim_{h \to 0} \frac{0}{h} = 0 & a \ne 0 \quad a > 0 \quad a + h > 0 \end{cases}$$

$*$ if $x \notin \mathbb{Z}$



$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{[x+h] - [x]}{h}$$

$$= \lim_{h \to 0} \frac{[x] - [x]}{h}$$

$0 < |h| < \min(x - [x] , [x] + 1 - x)$

$y = \sqrt{x}$ 정의 $x \ge 0$

$y' = \frac{1}{2\sqrt{x}}$ 정의 $x > 0$

[함수의 정의역은 원래함수의 정의역의 부분집합입니다.

$f(x) = (x^5+3) \cdot (x^3-2)$

$f'(x) = (x^5+3)'(x^3-2) + (x^5+3)\cdot(x^3-2)'$

$= 5x^4 \cdot (x^3-2) + (x^5+3)\cdot 3x^2$

* $f \circ g(x) = f(g(x))$

$$\frac{f\circ g(x+h) - f\circ g(x)}{h} = \frac{f(g(x+h)) - f(g(x))}{\underbrace{g(x+h)-g(x)}} \times \frac{g(x+h)-g(x)}{h}$$

$\rightarrow$ $g(x+h) - g(x) = 0$ 인 경우도 있다.

* $h(x) = \{g(x)\}^{n \in \mathbb{R}}$

$f(x) = x^n$ $\qquad f'(x) = nx^{n-1}$

$x^{\frac{n}{m}}$ $\quad x^{\sqrt{2}}$

$x^1 \quad x^{1.4} \quad x^{1.41} \quad x^{1.414} \cdots$

$$\{f\circ g(x)\}' = f'(g(x))\,g'(x)$$

$$= n\{g(x)\}^{n-1} g'(x)$$

연습문제 3-2

$\lim_{n\to\infty}\sqrt{\frac{1}{n}} = \lim_{h\to 0+}\sqrt{h}$ ($h = \frac{1}{n}$), $\boxed{\lim_{h\to 0-}\sqrt{h}\ (X)}$ ($h>0$) $\lim_{h\to 0}\sqrt{h}\ (X)$

연습문제 3-3

$$\lim_{x\to 0} x|x| = \begin{cases} \lim_{x\to 0+} x^2 = 0 \\ \lim_{x\to 0-} -x^2 = 0 \end{cases}$$

연속이므로

3-4 f: 기함수 $\qquad f'$: 우함수

f: 우함수 $\qquad f'$: 기함수

$\boxed{f'(0) = 0 \leftarrow x가\ 0에서\ 미분가능\ 하다면}$

3-5 * $\lim_{x\to a}|f(x)| = 0 \Leftrightarrow \lim_{x\to a} f(x) = 0$?

$$\begin{cases} \lim_{x\to a} |f(x)| = 0 & \forall \varepsilon > 0,\ \exists \delta > 0 \ \text{s.t.}\ 0 < |x-a| < \delta \Rightarrow \big||f(x)| - 0\big| < \varepsilon \\ & \big||f(x)|\big| = |f(x)| = |f(x) - 0| \\ \lim_{x\to a} f(x) = 0 & \forall \varepsilon > 0,\ \exists \delta > 0 \ \text{s.t.}\ 0 < |x-a| < \delta \Rightarrow |f(x) - 0| < \varepsilon \end{cases}$$

※ $f(x) \le g(x) \le h(x)$, $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L \Rightarrow \lim_{x\to a} g(x) = L$

$\lim_{x\to a} f(x) \le \lim_{x\to a} g(x) \le \lim_{x\to a} h(x)$ (포런 지앙)

3-9 $f(x) = (x-a)^m (x-b)^n$ $f'\left(\frac{mb+na}{m+n}\right) = 0$

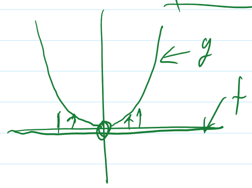
m, n 자연수

$\uparrow$ a, b 사이의 내분점 1509 장민준

3-10 $$\begin{cases} f(x) < g(x) \ (0 < |x-a| < \delta) \\ \exists \lim_{x\to a} f(x),\ \exists \lim_{x\to a} g(x) \end{cases} \Rightarrow \lim_{x\to a} f(x) \le \lim_{x\to a} g(x)$$

$\boxed{0 < \frac{1}{x}}$

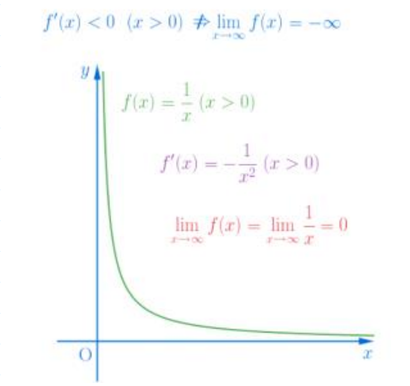
ex) $f(x) = 0$, $g(x) = x^2$, $a = 0$



3-11 $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ $(a_n \ne 0,\ a_i \in \mathbb{R})$

$\lim_{x\to\infty} f(x) = +\infty$ or $-\infty$ 임을 보여라.

※ $f'(x) < 0$ $(x > 0)$ $\Rightarrow \lim_{x\to\infty} f(x) = -\infty$? X

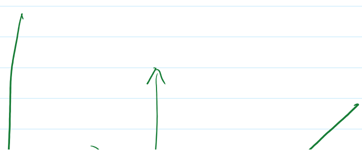


https://min7014.github.io/2019/2019082906.pdf

3-13 $f(x) = 4x^2 - 12x + 5$

$f \circ g(x) = f(x)$ $g(x)$?

$4(g(x))^2 - 12g(x) + 5 = 4x^2 \cdots$



$f \circ g(x) = f(x) \quad g(x)\,?$

$4\{g(x)\}^2 - 12\{g(x)\} + 5 = 4x^2 - 12x + 5$

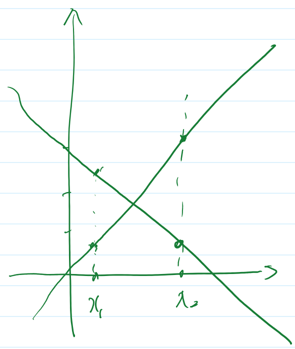
$4\{g(x)\}^2 - 12\{g(x)\} - 4x^2 + 12x = 0$

$\{g(x)\}^2 - 3\{g(x)\} - x(\underset{x-3}{x-3}) = 0$

$\{g(x) - x\}\{g(x) \overset{-x}{+} (x-3)\} = 0$

$\therefore\ g(x) = x\ ,\ 3-x$ ✗

$g(x) = x$ 또는 $g(x) = 3-x$



3-14 $g(x) = \begin{cases} f(x) & (x < a) \\ m - f(x) & (a \le x < b) \\ n + f(x) & (b \le x) \end{cases}$

$$\lim_{x\to a-} \frac{g(x) - g(a)}{x-a} = \lim_{x\to a-} \frac{f(x) - \cancel{\{m - f(a)\}}}{x-a} = \lim_{x\to a-} \frac{f(x) - f(a)}{x-a} = f'(a)$$

$$\lim_{x\to a+} \frac{g(x) - g(a)}{x-a} = \lim_{x\to a+} \frac{\{m - f(x)\} - \{m - f(a)\}}{x-a} = \lim_{x\to a+} \left\{-\frac{f(x) - f(a)}{x-a}\right\} = -f'(a)$$

$$\lim_{x\to a+} g(x) = \lim_{x\to a+} \{m - f(x)\} = m - f(a)$$

$$\lim_{x\to a-} g(x) = \lim_{x\to a-} f(x) = f(a)$$

3-15. f : 미분가능 $\not\Rightarrow$ f' : 연속

$y = x\sin\frac{1}{x}$

3-16 $f(x) = (x-a)^n Q(x) + R(x) \Rightarrow f'(x) = (x-a)^{n-1} Q_1(x) + R'(x)$

3-18 $\frac{d}{dx}\left(f(x)\right)$

곡선의 접선과 미분

2019년 9월 2일 월요일 오전 10:04

필수 4-1 (2) 기하적 의미
(1)

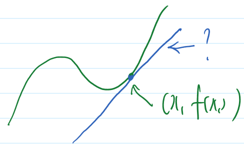
* 기울기 m, 한점 (x_1, y_1) 이면 $y - y_1 = m(x - x_1)$

* 함수 $y = f(x)$, $(x_1, f(x_1))$

접선: 기울기 $f'(x_1)$

$y - f(x_1) = f'(x_1)(x - x_1)$

$* \; f'(x_1)x - y + f(x_1) - f'(x_1)x_1 = 0$

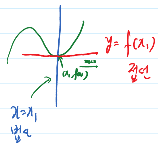


법선: i) $f'(x_1) \neq 0$ ii) $f'(x_1) = 0$

$m \times m' = -1$

기울기 $-\frac{1}{f'(x_1)}$

$y - f(x_1) = -\frac{1}{f'(x_1)}(x - x_1)$



$x = x_1$

$* \; x + f'(x_1)y - f'(x_1)f(x_1) - x_1 = 0$

$\begin{cases} ax + by + c = 0 \\ a'x + b'y + c' = 0 \end{cases}$ 수직 $\Longleftrightarrow aa' + bb' = 0$

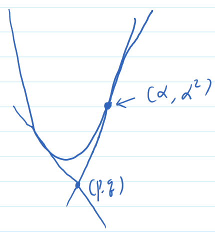
필수예제 4-4

* 한 점을 지나고 포물선과 접하는 직선의 개수에 대하여 논하여라.

$y = x^2$ (∵ 모든 포물선은 닮음이다)

(α, α^2) 의 접선의 방정식

$y - \alpha^2 = 2\alpha(x - \alpha)$



(p, q)를 지난다고 한다면

$q - \alpha^2 = 2\alpha(p - \alpha) = 2p\alpha - 2\alpha^2$

$\alpha^2 - 2p\alpha + q = 0$

$\begin{cases} 0\text{개}: \; D < 0 \quad p^2 < q \quad (p, q) \; (x^2 < y) \\ 1\text{개}: \; D = 0 \quad p^2 - q = 0 \quad (p, p^2) \; (x^2 = y) \\ 2\text{개}: \; D > 0 \quad p^2 > q \quad (p, q) \; (x^2 > y) \end{cases}$

* 포물선의 두 접선이 수직일때 교점을 모두 구하여라.

* 필수예제 4-5의 기하학적 의미?

기하적으로 계산값의 비가 일정한 선형함수 일는가?

* 필수예제 4-6 의 기하학적의미!

뉴턴근사 기하적으로 계산값의 비가 일정한 선형함수 일는가?

$[a,b] = \{x \mid a \le x \le b\}$

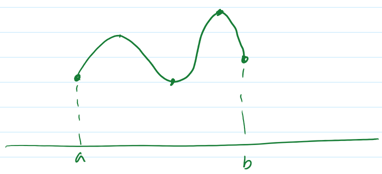
$(a,b) = \{x \mid a < x < b\}$

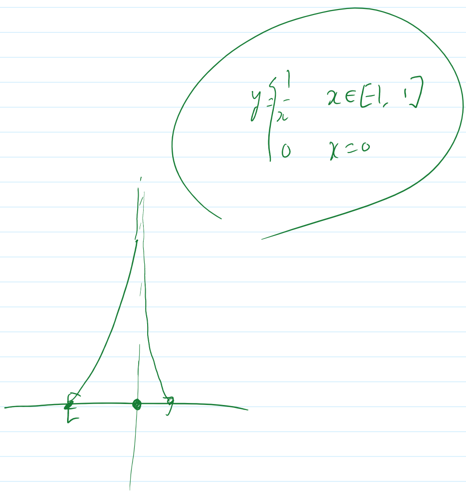
f가 $[a,b]$에서 연속 $\begin{cases} c \in (a,b) \quad \lim_{x\to c} f(x) = f(c) \\ \lim_{x\to a+} f(x) = f(a) \\ \lim_{x\to b-} f(x) = f(b) \end{cases}$

* 최대 최소 정리

f는 $[a,b]$에서 연속 $\Rightarrow$ 함수 f는 $[a,b]$에서 최댓값, 최솟값이 존재

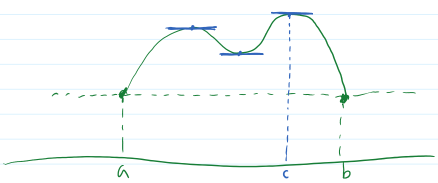
$$\begin{cases} \exists\, c_1 \in [a,b] \text{ s.t. } f(c_1) \ge f(x),\ \forall x \in [a,b] \\ \quad\quad \text{and} \\ \exists\, c_2 \in [a,b] \text{ s.t. } f(c_2) \le f(x),\ \forall x \in [a,b] \end{cases}$$





* 롤의 정리

$$\begin{cases} f\text{가 } [a,b]\text{에서 연속} \\ f\text{가 } (a,b)\text{에서 미분가능} \\ f(a) = f(b) \end{cases} \Rightarrow \exists\, c \in (a,b) \text{ s.t. } f'(c) = 0$$



proof) f가 $[a,b]$에 연속이므로

최대 최소 정리에 의해 최댓값 최솟값이 존재한다.

proof) f가 [a,b]에서 연속이므로
최대 최소 정리에 의해 최댓값 최솟값이 존재 한다.

Case 1, 최댓값과 최솟값은 f(a) $\quad . \; f(x) = f(a) \;,\; x \in [a,b]$

$f'(x) = 0 \;,\; x \in (a,b)$

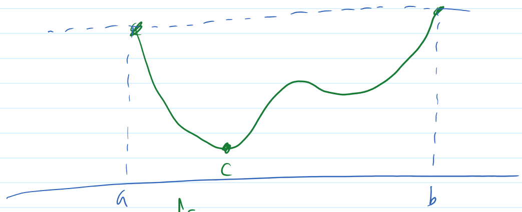
Case 2, Case 1이 아닌 경우

최소가 (a, b)에 있는 경우

$\exists c \in (a,b) \text{ s.t. } f(c) \le f(x) \,,\, x \in [a,b]$

$$\lim_{x \to c-} \frac{\overbrace{f(x)-f(c)}^{\ge 0}}{\underbrace{x-c}_{<0}} \le 0 \qquad \lim_{x \to c+} \frac{\overbrace{f(x)-f(c)}^{\ge 0}}{\underbrace{x-c}_{>0}} \ge 0$$

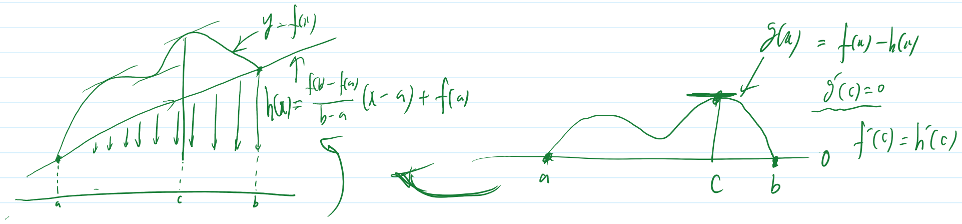
$f'(c)$



$\therefore f'(c) = 0$

* 평균값 정리

$$\begin{cases} f\text{가 } [a,b]\text{에서 연속} \\ f\text{가 } (a,b)\text{에서 미분가능} \end{cases} \Rightarrow \exists c \in (a,b) \text{ s.t. } f'(c) = \frac{f(b)-f(a)}{b-a}$$



증명) $g(x) = f(x) - \Big\{ \underbrace{(x-a) \cdot \frac{f(b)-f(a)}{b-a} + f(a)}_{h(x)} \Big\}$

$g(a) = 0 \qquad g(b) = 0$

$$\begin{cases} g\text{가 } [a,b]\text{에서 연속} \\ g\text{가 } (a,b)\text{에서 미분가능} \\ g(a) = g(b) \end{cases}$$

롤의 정리에 의해서
$\Rightarrow \exists c \in (a,b) \text{ s.t. } g'(c) = 0$

$g'(x) = f'(x) - \frac{f(b)-f(a)}{b-a}$

$g'(c) = 0$

$f'(c) = \frac{f(b)-f(a)}{b-a}$

$\therefore\ \exists c \in (a,b)$ s.t. $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

정리 4-7

* $\begin{cases} f\text{가 } [a,b]\text{에서 연속} \\ f'\text{이 } (a,b)\text{에서 } f'=0 \end{cases} \Rightarrow f$가 $[a,b]$에서 $f(x)=f(a)$

proof) $\forall x \in (a,b)$

$\begin{cases} f\text{가 } [a,x]\text{에서 연속} \\ f\text{가 } (a,x)\text{에서 미분가능 } (\because f'(t)=0,\ t\in(a,b)) \end{cases}$

평균값 정리에 의해서

$\exists c \in (a,x)$ s.t. $f'(c) = \dfrac{f(x)-f(a)}{x-a}$

$\begin{cases} x-a\neq 0 \ (\because c\in(a,x)) \\ f'(c)=0 \end{cases} \Rightarrow f(x)-f(a)=0$

$f(x)=f(a),\ x\in(a,b)$ ... ①

$f(b) = \lim_{x\to b-} f(x)$ ($\because$ f가 $[a,b]$에서 연속)

$= \lim_{x\to b-} f(a) = f(a)$ .. ②

①, ②에 의해서

$f(x)=f(a),\ x\in[a,b]$

* 연습문제 4-5 조건이 겹치는데

* 연습문제 4-10

왜 포물선은 쓰지?

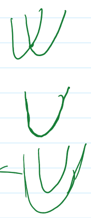
* 두 이차함수의 교점에 대해서

$\begin{cases} y=a_1x^2+b_1x+c_1 & (a_1\neq 0) \\ y=a_2x^2+b_2x+c_2 & (a_2\neq 0) \end{cases}$

$(a_1-a_2)x^2+(b_1-b_2)x+c_1-c_2=0$

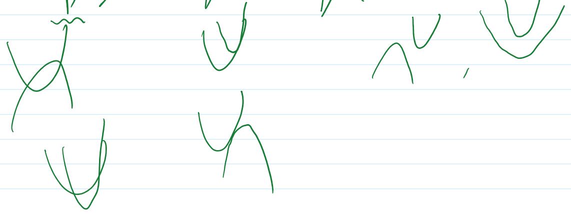
I) $a_1-a_2=0$

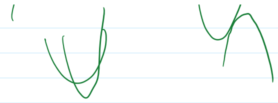
$(b_1-b_2)x+c_1-c_2 \Leftarrow ax=b$ $\begin{cases} a\neq 0 \\ a=0,\ b=0 \\ a=0,\ b\neq 0 \end{cases}$



II) $a_1-a_2\neq 0$

$D>0$ $\quad D=0$ $\quad D<0$





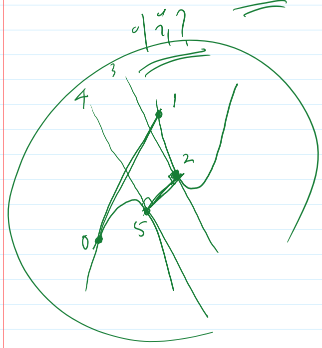
★ 연습문제 4-13

i) $\forall a,\ f(a) \ge m$

ii) $\exists a_0$ s.t. $f(a_0) = m$

$\Rightarrow$ 최소 m

★ 연습문제 4-14 이유!

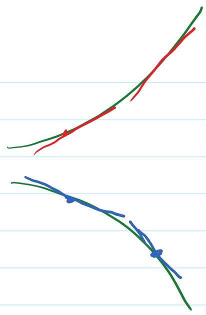


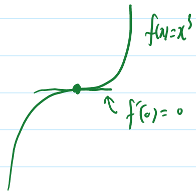
$$d(\alpha, \beta) = \min_{\substack{P \in \alpha \\ Q \in \beta}} d(P, Q)$$

극대, 극소와 미분

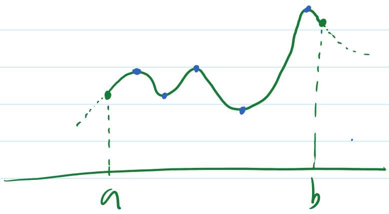
2019년 9월 3일 화요일 오후 3:55

* 증가 $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$
* 감소 $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$





극대

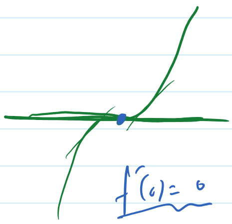


$f'(c_1)=0$

극소

$f(x) = x^3$

$f'(x) = 3x^2$

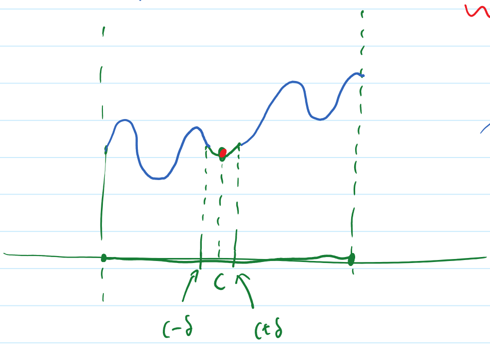


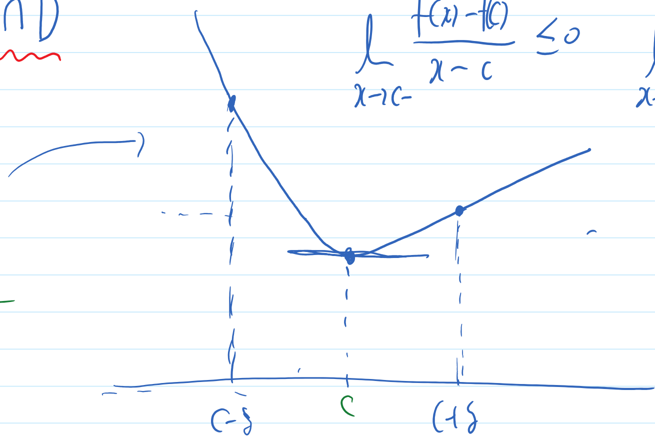
$f'(0) = 0$

f가 정의역 D에서 정의된 함수일 때,

* c∈D 에서 극솟값을 갖는다.

$\exists \delta > 0 \text{ s.t. } f(c) \le f(x),\ x \in (c-\delta, c+\delta) \cap D$

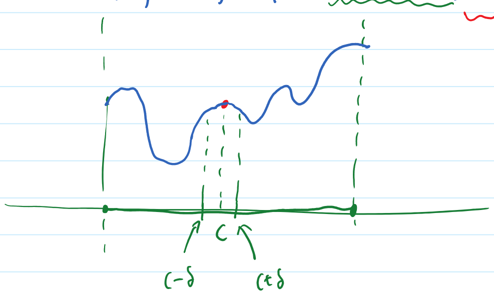


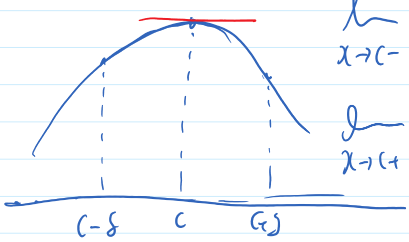


$\lim_{x\to c^-} \frac{f(x)-f(c)}{x-c} \le 0$ $\quad \lim_{x\to c^+} \frac{f(x)-f(c)}{x-c} \ge 0$

* c∈D 에서 극댓값을 갖는다.

$\exists \delta > 0 \text{ s.t. } f(c) \ge f(x),\ x \in (c-\delta, c+\delta) \cap D$

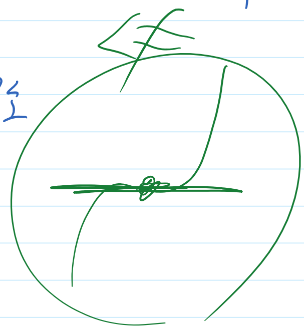




$\lim_{x\to c^-} \frac{f(x)-f(c)}{x-c} \ge 0$

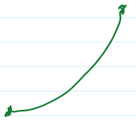
$\lim_{x\to c^+} \frac{f(x)-f(c)}{x-c} \le 0$

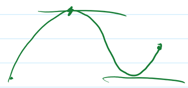
$$* \begin{cases} f\text{가 } (a,b)\text{에서 미분가능} \\ c \in (a,b) \\ f\text{는 } c\text{에서 극대 또는 극소} \end{cases} \Rightarrow f'(c) = 0$$

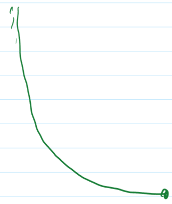


최대 최소와 미분

2019년 9월 3일 화요일 오후 4:01







$f(x) = \begin{cases} \frac{1}{x}, & (x \in (0,1]) \\ 1, & x = 0 \end{cases}$

* f가 [a,b]에서 연속 ⇒ 최댓값 $= \max_{x \in [a,b]} \{f(x)\}$ 최솟값 $= \min_{x \in [a,b]} \{f(x)\}$

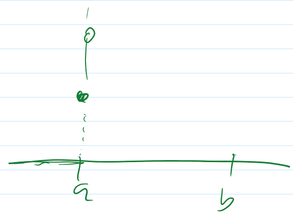
* f가 [a,b]에서 연속 ⇒ $R = \{x \mid x$에서 극값을 갖는다., $x \in (a,b)\}$

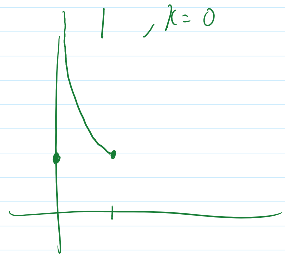
최댓값 $= \max_{x \in R \cup \{a,b\}} \{f(x)\}$ 최솟값 $= \min_{x \in R \cup \{a,b\}} \{f(x)\}$

* f가 [a,b]에서 연속, f가 (a,b)에서 미분가능 ⇒ $R = \{x \mid f'(x) = 0, x \in (a,b)\}$

최댓값 $= \max_{x \in R \cup \{a,b\}} \{f(x)\}$

최솟값 $= \min_{x \in R \cup \{a,b\}} \{f(x)\}$





함수의 나눗셈의 미분

2019년 10월 22일 화요일 오전 9:12

$$\text{※}\quad f(x) = \frac{h(x)}{g(x)} = h(x) \cdot \frac{1}{g(x)}$$

$$\underline{f'(x)} = h'(x) \cdot \frac{1}{g(x)} + h(x) \cdot \left\{ - \frac{g'(x)}{\{g(x)\}^2} \right\} \quad (\Leftarrow \text{합성함수})$$

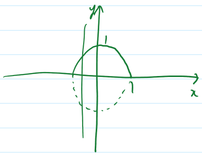
$$= \underline{\frac{h'(x)g(x) - h(x)g'(x)}{\{g(x)\}^2}}$$

* 음함수 $F(x, y) = 0$

ex) $\boxed{x^2+y^2-1=0}$ $x^2+y^2=1$

$F(x,y)=x^2+y^2-1$

$\begin{cases} -1 \le x \le 1 \\ 0 \le y \end{cases}$



ex) $y=f(x)$

$y-f(x)=0$

$F(x,y)=y-f(x)$

* 음함수 미분

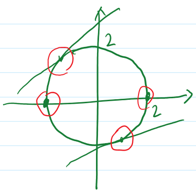
미분은 x_0 값 근처에서만 생각하면 되므로 적당한 (x_0,y_0)의 근방에서 그래프가 y는 x에 관한 함수로 볼 수 있으므로 합성함수 미분법을 적용할 수 있다.

$y=f(x)$

$y^2=\{f(x)\}^2$

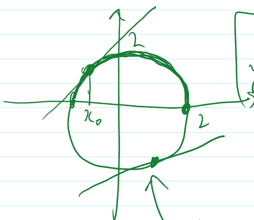
$(y^2)'=2f(x)\cdot f'(x)$

$x^2+y^2-4=0 \longrightarrow 2x+2yy'=0$



$2yy'=-2x$

$y'=-\frac{x}{y}$ $(y\neq 0)$



$y=\sqrt{4-x^2}$

$y'=\frac{-2x}{2\sqrt{4-x^2}}=\frac{-x}{\sqrt{4-x^2}}=-\frac{x}{y}$

$y=-\sqrt{4-x^2}$

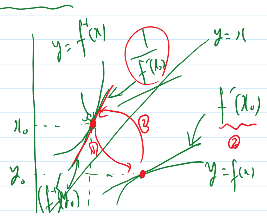
$y'=-\frac{-2x}{2\sqrt{4-x^2}}=\frac{-x}{-\sqrt{4-x^2}}=-\frac{x}{y}$

* 역함수의 미분법

$y=f(x)$

$x=f^{-1}(y)$

$(f^{-1}(y))'=(f^{-1})'(y)=\frac{1}{f'(x)}$

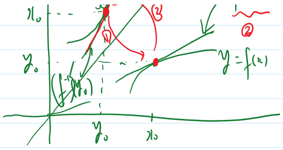


y에 관한

$(f^{-1})'(y_0)=\frac{1}{f'(x_0)}$

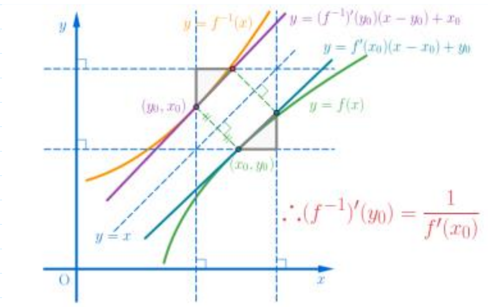
x에 관한 미분

$f^{-1}(x)$



$f'(x_0)$

x에 관한 미분



https://min7014.github.io/math20191016001.html

* 매개변수 함수의 미분

ex) $y=x$ $\quad \begin{cases} x=t \\ y=t \end{cases}$

ex) $y=x^2$ $\quad \begin{cases} x=t \\ y=t^2 \end{cases}$ $\quad \begin{cases} x=at \\ y=a^2t^2 \end{cases}$ $(a\neq 0)$

* $\begin{cases} x=f(t) \\ y=g(t) \end{cases}$

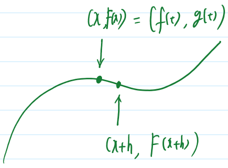
$$\left.\frac{dy}{dx}\right|_{x=x_0}=\frac{\left.\frac{dy}{dt}\right|_{t=f^{-1}(x_0)}}{\left.\frac{dx}{dt}\right|_{t=f^{-1}(x_0)}}=\left.\frac{g'(t)}{f'(t)}\right|_{t=f^{-1}(x_0)}$$

* $y=f(x)$

$\begin{cases} x=t \\ y=f(t) \end{cases}$

$y=F(x)$

$\begin{cases} x=f(t) \\ y=g(t) \end{cases}$



$h=f(t+l)-f(t)\longrightarrow 0$ as $l\longrightarrow 0$

$$\frac{dy}{dx}=\lim_{h\to 0}\frac{F(x+h)-F(x)}{h}=\lim_{h\to 0}\frac{F(x+h)-F(x)}{x+h-x}$$

$$=\lim_{l\to 0}\frac{g(t+l)-g(t)}{f(t+l)-f(t)}$$

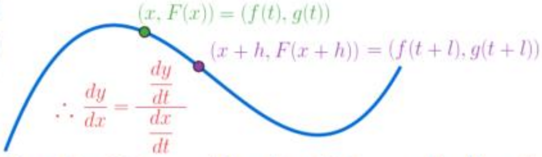
$$=\lim_{l\to 0}\frac{\frac{g(t+l)-g(t)}{l}}{\frac{f(t+l)-f(t)}{l}}=\frac{g'(t)}{f'(t)}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\therefore \frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

https://min7014.github.io/math20191020001.html

$y = F(x)$

$\begin{cases} x = f(t) \\ y = g(t) \end{cases}$



$\therefore \dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{F(x+h) - F(x)}{h} = \lim_{h \to 0} \frac{F(x+h) - F(x)}{x+h-x} = \lim_{l \to 0} \frac{g(t+l) - g(t)}{f(t+l) - f(t)}$$

$$= \lim_{l \to 0} \frac{\frac{g(t+l) - g(t)}{l}}{\frac{f(t+l) - f(t)}{l}} = \frac{g'(t)}{f'(t)} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

※ 이계도함수에 대한 매개변수로의 표현

$\begin{cases} x = x(t) \\ y = y(t) \end{cases}$ $\boxed{\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}}$ $\dfrac{d^2y}{dx^2} = \cancel{\dfrac{\frac{d^2y}{dt^2}}{\frac{d^2x}{dt^2}}}$

$$\frac{d^2y}{dx^2} = \left(\frac{d}{dx}\right)^2 y = \frac{d}{dx}\left(\frac{d}{dx}y\right) = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x) \cdot g(x) - f(x)g'(x)}{\{g(x)\}^2}$$

$Y = \dfrac{dy}{dx}$

$$= \frac{dY}{dx} = \frac{\frac{dY}{dt}}{\frac{dx}{dt}} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt}\left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)}{\frac{dx}{dt}}$$

$$= \left\{\frac{\frac{d^2y}{dt^2} \cdot \frac{dx}{dt} - \frac{dy}{dt} \cdot \frac{d^2x}{dt^2}}{\left(\frac{dx}{dt}\right)^2}\right\} \div \left(\frac{dx}{dt}\right) = \frac{\frac{d^2y}{dt^2} \cdot \frac{dx}{dt} - \frac{dy}{dt} \cdot \frac{d^2x}{dt^2}}{\left(\frac{dx}{dt}\right)^3}$$

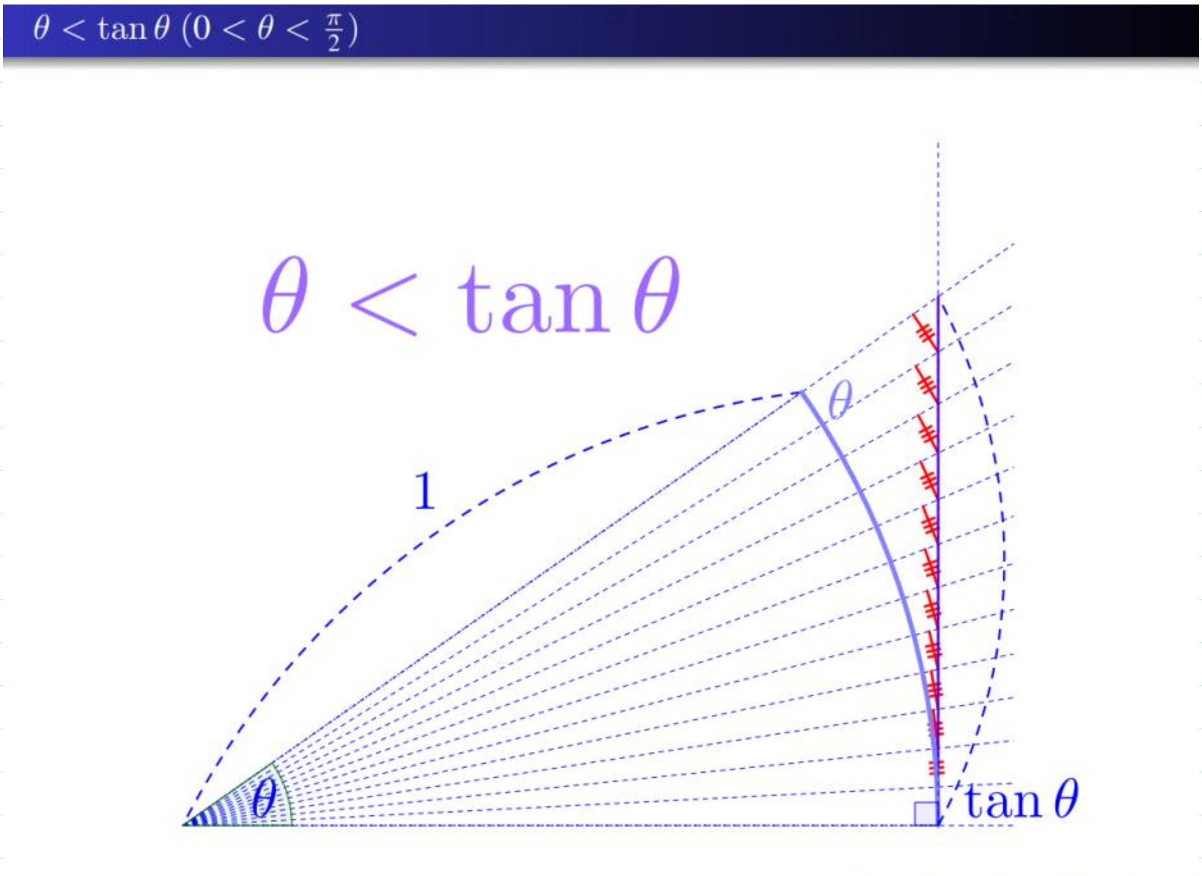
삼각함수의 미분

2019년 10월 21일 월요일 오후 1:28

http://min7014.iptime.org/origin/%EC%88%98%ED%95%99%EC%9E%90%EB%A3%8C%EC%8B%A4/%EA%B3%A0%EB%93%B1%ED%95%99%EA%B5%90/%EC%88%98%ED%95%99II/%EC%82%BC%EA%B0%81%ED%95%A8%EC%88%98%EC%9D%98%20%EB%8D%A7%EC%85%88%EC%A0%95%EB%A6%AC.htm

http://min7014.iptime.org/origin/%EC%88%98%ED%95%99%EC%9E%90%EB%A3%8C%EC%8B%A4/%EA%B3%A0%EB%93%B1%ED%95%99%EA%B5%90/%EC%88%98%ED%95%99II/%EC%82%BC%EA%B0%81%ED%95%A8%EC%88%98%EC%9D%98%20%EA%B3%B5%EC%8B%9D%EA%B3%BC%20%EB%B0%A9%EC%A0%95%EC%8B%9D.htm

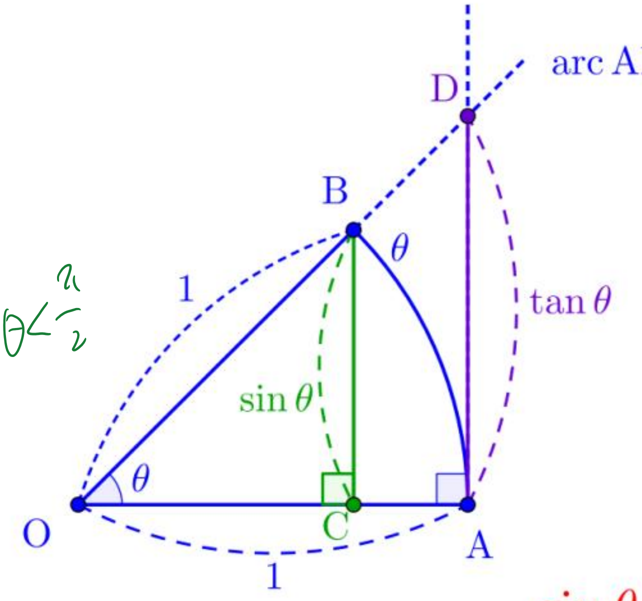
https://min7014.github.io/2019/2016040401.pdf#toolbar=0&view=Fit&scrollbar=0



https://min7014.github.io/2019/2016032701.pdf#toolbar=0&view=Fit&scrollbar=0

$\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$

$0<\theta<\frac{\pi}{2}$



arc AB $<\overline{\mathrm{AD}}$ $\qquad$ $\overline{\mathrm{BC}}<$ arc AB

$\theta<\tan\theta \qquad \sin\theta<\theta$

$\theta<\tan\theta$

$\theta<\frac{\sin\theta}{\cos\theta}$

$$\cos\theta<\frac{\sin\theta}{\theta}<1$$

$$\lim_{\theta\to 0^+}\cos\theta=1$$

$$\lim_{\theta\to 0^+}\frac{\sin\theta}{\theta}=1$$

$(\because \frac{\sin\theta}{\theta}$ is even.)

$f(x)=f(-x)$

$$\therefore \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$$

Min Eun Gi : https://www.facebook.com/mineungimath

* $y=\sin x$

$$\frac{dy}{dx}=\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}=\lim_{h\to 0}\frac{\overbrace{\underbrace{\sin x\cos h+\cos x\sin h}_{}}^{\sin(x+h)}-\sin x}{h}$$

덧셈 정리

$$=\lim_{h\to 0}\left\{\cos x\cdot\underbrace{\frac{\sin h}{h}}_{\to 1}+\sin x\cdot\underbrace{\frac{\cos h-1}{h}}_{\to 0}\right\}=\cos x$$

[

$$\lim_{h\to 0}\frac{\cos h-1}{h}=\lim_{h\to 0}\frac{(\cos h-1)\times(\cos h+1)}{h\times(\cos h+1)}=\lim_{h\to 0}\frac{\overbrace{\cos^2 h-1}^{-\sin^2 h}}{h(\cos h+1)}$$

$$=\lim_{h\to 0}\underbrace{\frac{\sin h}{h}}_{\to 1}\cdot\underbrace{(-\sin h)}_{\to 0}\cdot\frac{1}{\underbrace{\cos h}_{\to 1}+1}=0$$

1 0 $\frac{1}{2}$

* $y = \cos x = \sin\left(x + \frac{\pi}{2}\right)$

$$\frac{dy}{dx} = \cos\left(x + \frac{\pi}{2}\right) \cdot 1 = -\sin x$$

* $y = \tan x = \dfrac{\sin x}{\cos x}$

$$\boxed{\sin x \quad \cos x \quad \tan x \quad \csc x \quad \sec x \quad \cot x}$$

지수함수의 미분

2019년 10월 21일 월요일 오후 2:08

* $y = e^x$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \to 0} e^x \cdot \frac{e^h - 1}{h} = e^x$$

$$\lim_{h \to 0} \frac{e^h - 1}{h} = 1$$

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$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\{e^{f(x)}\}' = e^{f(x)} \cdot f'(x)$$

* $y = a^x = e^{x \ln a}$

$$\frac{dy}{dx} = e^{x \ln a} \cdot \ln a = a^x \ln a$$

* $y = \ln x \quad x = e^y \quad 1 = e^y \cdot y' \quad y' = \frac{1}{e^y} = \frac{1}{x}$

$$\frac{dy}{dx} = \frac{1}{x}$$

* $y = \log_a x = \frac{\ln x}{\ln a}$

$$\frac{dy}{dx} = \frac{1}{\ln a} \cdot \frac{1}{x} = \frac{1}{x \ln a}$$

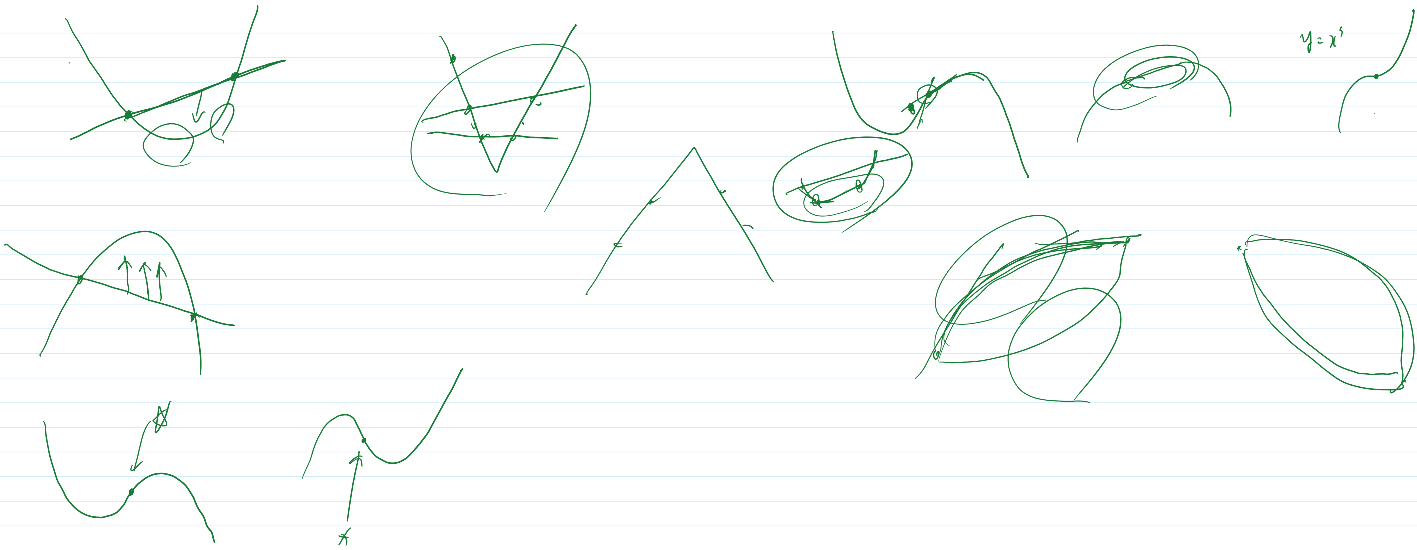
오목과 볼록

2019년 10월 28일 월요일 오후 1:50

오목:

볼록:

변곡점:



* 코시의 평균값 정리

($g(x)=x$ 이면 평균값 정리)

$[a,b]$ 에서 f, g 가 연속
(a,b) 에서 f, g 가 미분가능
$\forall x \in (a,b),\ g'(x) \neq 0$

$\Rightarrow \exists\, c \in (a,b)$ s.t. $\dfrac{f'(c)}{g'(c)} = \dfrac{f(b)-f(a)}{g(b)-g(a)}$

pf) $[a,b]$ 에서 g는 연속
(a,b) 에서 g는 미분가능

$\underset{\text{평균값 정리}}{\Longrightarrow} \exists\, c_1 \in (a,b)$ s.t. $\dfrac{g(b)-g(a)}{b-a} = g'(c_1)$

$g(b)-g(a) = (b-a)\,g'(c_1) \neq 0 \quad \therefore\ g(a) \neq g(b)$

Let $F(x) = f(x) - f(a) - \dfrac{f(b)-f(a)}{g(b)-g(a)}\{g(x)-g(a)\}$ $(\because g(a) \neq g(b))$

$[a,b]$ 에서 F는 연속
(a,b) 에서 F는 미분가능
$F(a) = F(b) = 0$

$\Rightarrow \exists\, c_2 \in (a,b)$ s.t. $\dfrac{F(b)-F(a)}{b-a} = F'(c_2)$

$$F'(x) = f'(x) - \frac{f(b)-f(a)}{g(b)-g(a)} \cdot g'(x)$$

$$\therefore F'(c_2) = f'(c_2) - \frac{f(b)-f(a)}{g(b)-g(a)} \cdot g'(c_2) = 0 \quad (\because F(a)=F(b)=0)$$

$$\frac{f'(c_2)}{g'(c_2)} = \frac{f(b)-f(a)}{g(b)-g(a)} \quad (\because g'(c_2) \neq 0)$$

* 로피탈의 정리

ex) $|x|$

a를 포함하는 열린 구간 I 에 대하여
f, g는 $I - \{a\}$ 에서 미분가능
f, g는 I 에서 연속
$f(a) = g(a) = 0$
$\forall x \in I - \{a\},\ g'(x) \neq 0$
$\exists \lim_{x\to a} \dfrac{f(x)}{g(x)}$

$\Rightarrow \lim_{x\to a} \dfrac{f(x)}{g(x)} = \lim_{x\to a} \dfrac{f'(x)}{g'(x)}$

$$\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{\cos x}{1} = 1$$

증명: I) $x > a$

$[a,x]$ 에서 f, g는 연속
(a,x) 에서 f, g는 미분가능
$\forall t \in (a,x),\ g'(t) \neq 0$

$\underset{\text{코시의 평균값 정리}}{\Rightarrow} \exists\, c_x \in (a,x)$ s.t. $\dfrac{f(x)-f(a)}{g(x)-g(a)} = \dfrac{f'(c_x)}{g'(c_x)}$

$$\lim_{x\to a+} \frac{f(x)}{g(x)} = \lim_{x\to a+} \frac{f'(c_x)}{g'(c_x)} = \underbrace{\lim_{c_x\to a+} \frac{f'(c_x)}{g'(c_x)} = \lim_{x\to a+}}_{\text{자리바꿈}} \frac{f'(x)}{g'(x)}$$

ⅱ) $x<a$ similary

$$\lim_{x\to a} f(x) = \infty \quad \lim_{x\to a} g(x) = \infty \qquad F(x) = \begin{cases} 0 & , x=a \\ \frac{1}{f(x)} & , x\neq a \end{cases} \qquad G(x) = \begin{cases} 0 & , x=a \\ \frac{1}{g(x)} & , x\neq a \end{cases}$$

$$\lim_{x\to a} \frac{1}{f(x)} = 0 \quad \lim_{x\to a} \frac{1}{g(x)} = 0$$

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{\frac{1}{g(x)}}{\frac{1}{f(x)}} = \lim_{x\to a} \frac{-\frac{g'(x)}{\{g(x)\}^2}}{-\frac{f'(x)}{\{f(x)\}^2}} = \lim_{x\to a} \left[\left\{\frac{f(x)}{g(x)}\right\}^2 \cdot \frac{g'(x)}{f'(x)}\right]$$

$$= \left\{\lim_{x\to a} \frac{f(x)}{g(x)}\right\}^2 \cdot \lim_{x\to a} \frac{g'(x)}{f'(x)}$$

* $[\forall x,\ f(a-x)=f(a+x)] \Rightarrow [\,f$는 $x=a$에 대칭$\,]$

함수 $y=f(x)$, 축 $x=a$

함수 $y=g(x)=f(x+a)$, 축 $x=0$

x축 방향으로 $-a$ 만큼 평행이동

$g(x)=f(x+a)=f(a+x)=f(a-x)=f((-x)+a)=g(-x)$

$g(x)=g(-x)$

∴ g는 $x=0$에 대칭

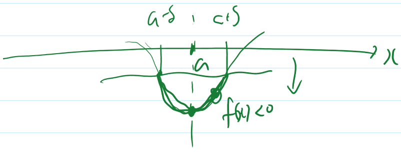
f는 $x=a$에 대칭

평행이동에 대칭성은 바뀌지 않으므로 대칭축만 평행이동 시킨다.

* $[\forall x\in(a,b),\ f'(x)<0] \Rightarrow f$는 (a,b)에서 감소 함수
* $[\forall x\in(a,b),\ f'(x)>0] \Rightarrow f$는 (a,b)에서 증가 함수

⇐ 평균값 정리

* $[f$가 a에서 연속 $\wedge\ f(a)<0] \Rightarrow [\exists\delta>0$ s.t. $\forall x\in(a-\delta,a+\delta) \Rightarrow f(x)<0]$



$\forall\varepsilon>0,\ \exists\delta>0$ s.t. $x\in(a-\delta,a+\delta) \Rightarrow |f(x)-f(a)|<\varepsilon$

$f(a)<0$

$-\varepsilon < f(x)-f(a) < \varepsilon$

$-\varepsilon + f(a) < f(x) < \varepsilon + f(a) = \dfrac{|f(a)|}{2} + f(a) = -\dfrac{f(a)}{2} + f(a) = \dfrac{f(a)}{2} < 0$

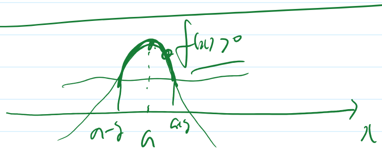
Let $\varepsilon = \dfrac{|f(a)|}{2}$

$\boxed{\exists\delta \text{ s.t. } x\in(a-\delta,a+\delta) \Rightarrow f(x)<0}$

절이 < 절댓값제

안나옴

* $[f$가 a에서 연속 $\wedge\ f(a)>0] \Rightarrow [\exists\delta>0$ s.t. $\forall x\in(a-\delta,a+\delta) \Rightarrow f(x)>0]$



* f'가 a에서 연속, $f'(a)<0$ $\Rightarrow$ $[\exists\delta>0 \text{ s.t. } (\forall x\in(a-\delta, a+\delta),\ f'(x)<0)]$

$\Rightarrow$ $[\exists\delta>0 \text{ s.t. }$ f가 $(a-\delta, a+\delta)$에서 감소$]$

* f''가 a에서 연속, $f''(a)<0$ $\Rightarrow$ $[\exists\delta>0 \text{ s.t. }$ f'가 $(a-\delta, a+\delta)$에서 감소$]$

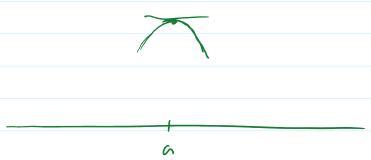
* f'가 a에서 연속, $f'(a)>0$ $\Rightarrow$ $[\exists\delta>0 \text{ s.t. } x\in(a-\delta, a+\delta) \Rightarrow f'(x)>0]$

$\Rightarrow$ $[\exists\delta>0 \text{ s.t. }$ f가 $(a-\delta, a+\delta)$에서 증가$]$

* f''가 a에서 연속, $f''(a)>0$ $=$ $[\exists\delta>0 \text{ s.t }$ f'가 $(a-\delta, a+\delta)$에서 증가$]$

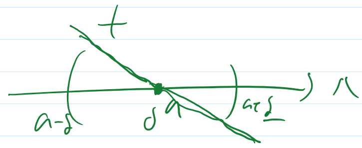
* $f''(x)$에 의한 극값의 판정

$[f'(a)=0 \wedge f''(a)<0 \wedge f''$는 a에서 연속$]$ $\Rightarrow$ $f(x)$는 a에서 극대



$\exists\delta>0 \text{ s.t }$ f'가 $(a-\delta, a+\delta)$에서 감소

	$a-\delta$	$\cdots$	a	$\cdots$	$a+\delta$
f'		$+$	0	$-$	



$[f'(a)=0 \wedge f''(a)>0 \wedge f''$는 a에서 연속$]$ $\Rightarrow$ $f(x)$는 a에서 극소

$\Rightarrow$ $[f'(a)=0 \wedge (\exists\delta>0 \text{ s.t }$ f'가 $(a-\delta, a+\delta)$에서 증가$)]$