함수
(Function)

## Function

```
 Start
End
```


## Function

```
* Start > End
```


## Definition (Function)

```
* Start
    \ End
```

Definition (Function)
A function $f$

```
* Start
     End
```

Definition (Function)

## A function $f$ is

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 Start
    D End
```

Definition (Function)

## A function $f$ is a rule

## Definition (Function)

A function $f$ is a rule that assigns to each element $x$

## Definition (Function)

A function $f$ is a rule that assigns to each element $x$ in a set $D$

## Definition (Function)

A function $f$ is a rule that assigns to each element $x$ in a set $D$ exactly one element,

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## Function

$>$ Home $>$ Start $>$ End

## Function

$\rightarrow$ Home $\xlongequal{\perp}$ Start $\triangle$ End
A function $f$
$\rightarrow$ Home $\rightarrow$ Start $\rightarrow$ End
A function $f$ from X

A function $f$ from X to Y

A function $f$ from X to Y is a subset

A function $f$ from X to Y is a subset of the Cartesian product $\mathrm{X} \times \mathrm{Y}$

A function $f$ from X to Y is a subset of the Cartesian product $\mathrm{X} \times \mathrm{Y}$ subject

A function $f$ from X to Y is a subset of the Cartesian product $\mathrm{X} \times \mathrm{Y}$ subject to the following condition:

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Every element

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Every element of X is the first component

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Every element of X is the first component of one and only one

- Start
$\rightarrow$ End
A function $f$ from X to Y is a subset of the Cartesian product $\mathrm{X} \times \mathrm{Y}$ subject to the following condition:
Every element of X is the first component of one and only one ordered pair

Start $>$ End
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Every element of X is the first component of one and only one ordered pair in the subset.

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In other words,

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## , Home <br> - Start $>$ End

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## , Home <br> - Start $>$ End

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## , Home <br> - Start $>$ End

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## , Home

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In other words, for every $x$ in X there is exactly one element $y$ in Y such that the ordered pair $(x, y)$ is contained in the subset defining the function $f$.

## Function

## Function

## Home <br> Start <br> End

$\mathrm{X} \times \mathrm{Y}$

## Function

```
    Home > Start > End
X }\times\textrm{Y}
```


## Function

```
\bulletHome > Start > End
X}\times\textrm{Y}=
```


## Function

```
    >Home > Start > End
X}\times\textrm{Y}={(x,y
```


## Function

$>$ Home $>$ Start $>$ End
$X \times Y=\{(x, y) \mid$

## Function

$>$ Home $>$ Start $>$ End
$X \times Y=\{(x, y) \mid x \in X$

## Function

```
> Home > Start > End
X}\times\textrm{Y}={(x,y)|x\in\textrm{X},y\in\textrm{Y}
```


## Function

Home $>$ Start $>$ End
$X=\{(x, y) \mid x \in X, y \in Y$

## Function

$\rightarrow$ Home $\rightarrow$ Start $\rightarrow$ End
$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product

## Function

$\rightarrow$ Home $\rightarrow$ Start $\rightarrow$ End
$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$

## Function

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\Home > Start > End
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$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$

## Function

## $\rightarrow$ Home $\rightarrow$ Start $\rightarrow$ End

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function

## Function

## $\rightarrow$ Home $\rightarrow$ Start $\rightarrow$ End

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X

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## $\rightarrow$ Home $\rightarrow$ Start $\rightarrow$ End

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

## Function

```
Home > Start > End
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$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y . !

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```
Home > Start > End
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$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .
$\left\{\begin{array}{l}f \subset \\ \end{array}\right.$

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## $\rightarrow$ Home $\rightarrow$ Start $\rightarrow$ End

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .
$\left\{\begin{array}{l}f \subset X \times Y \\ \end{array}\right.$

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## $\rightarrow$ Home $>$ Start $\rightarrow$ End

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
f \subset \mathrm{X} \times \mathrm{Y} \\
\forall x \in \mathrm{X}
\end{array}\right.
$$

## Function

## $\rightarrow$ Home $>$ Start $\rightarrow$ End

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

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\left\{\begin{array}{l}
f \subset \mathrm{X} \times \mathrm{Y} \\
\forall x \in \mathrm{X},
\end{array}\right.
$$

## Function

## $\rightarrow$ Home $\rightarrow$ Start $\rightarrow$ End

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
f \subset \mathrm{X} \times \mathrm{Y} \\
\forall x \in \mathrm{X}, \exists!y \in \mathrm{Y}
\end{array}\right.
$$

## Function

## $\rightarrow$ Home $\rightarrow$ Start $\rightarrow$ End

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
f \subset \mathrm{X} \times \mathrm{Y} \\
\forall x \in \mathrm{X}, \exists!y \in \mathrm{Y} \text { s.t. }
\end{array}\right.
$$

## Function

## $\rightarrow$ Home $\rightarrow$ Start $\rightarrow$ End

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
f \subset \mathrm{X} \times \mathrm{Y} \\
\forall x \in \mathrm{X}, \exists!y \in \mathrm{Y} \text { s.t. }(x, y) \in f
\end{array}\right.
$$

## Function

## $\rightarrow$ Home $\rightarrow$ Start $>$ End

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

## Function

## $\rightarrow$ Home $\rightarrow$ Start $>$ End

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
f \subset \mathrm{X} \times \mathrm{Y} \\
\forall x \in \mathrm{X}, \exists!y \in \mathrm{Y} \text { s.t. }(x, y) \in f\left\{\begin{array}{l}
\forall x \in \mathrm{X}, \exists y \in \mathrm{Y}
\end{array}, \$\right. \text {. }
\end{array}\right.
$$

## Function

## $\rightarrow$ Home $>$ Start $>$ End

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\begin{aligned}
& \{f \subset \mathrm{X} \times \mathrm{Y} \\
& \{x \in \mathrm{X}, \exists!y \in \mathrm{Y} \text { s.t. }(x, y) \in f\{\forall x \in \mathrm{X}, \exists y \in \mathrm{Y} \text { s.t. }(x, y) \in f
\end{aligned}
$$

## $\rightarrow$ Home $>$ Start $>$ End

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\begin{aligned}
& f f \subset \mathrm{X} \times \mathrm{Y} \\
& \left\{\begin{array}{l}
\forall \mathrm{X}, \exists!y \in \mathrm{Y} \text { s.t. }(x, y) \in f\left\{\begin{array}{l}
\forall x \in \mathrm{X}, \exists y \in \mathrm{Y} \text { s.t. }(x, y) \in f \\
\left(x, y_{1}\right) \in f
\end{array}\right.
\end{array}\right.
\end{aligned}
$$

## $\rightarrow$ Home $>$ Start $>$ End

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
f \subset \mathrm{X} \times \mathrm{Y} \\
\forall x \in \mathrm{X}, \exists!y \in \mathrm{Y} \text { s.t. }(x, y) \in f\left\{\begin{array}{l}
\forall x \in \mathrm{X}, \exists y \in \mathrm{Y} \text { s.t. }(x, y) \in f \\
\left(x, y_{1}\right) \in f \text { and }\left(x, y_{2}\right) \in f
\end{array}\right.
\end{array}\right.
$$

## $\rightarrow$ Home $>$ Start $>$ End

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
f \subset \mathrm{X} \times \mathrm{Y} \\
\forall x \in \mathrm{X}, \exists!y \in \mathrm{Y} \text { s.t. }(x, y) \in f\left\{\begin{array}{l}
\forall x \in \mathrm{X}, \exists y \in \mathrm{Y} \text { s.t. }(x, y) \in f \\
\left(x, y_{1}\right) \in f \text { and }\left(x, y_{2}\right) \in f \Rightarrow
\end{array}\right.
\end{array}\right.
$$

## $\rightarrow$ Home $\perp$ Start $\rightarrow$ End

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
f \subset \mathrm{X} \times \mathrm{Y} \\
\forall x \in \mathrm{X}, \exists!y \in \mathrm{Y} \text { s.t. }(x, y) \in f\left\{\begin{array}{l}
\forall x \in \mathrm{X}, \exists y \in \mathrm{Y} \text { s.t. }(x, y) \in f \\
\left(x, y_{1}\right) \in f \text { and }\left(x, y_{2}\right) \in f \Rightarrow y_{1}=y_{2}
\end{array}\right.
\end{array}\right.
$$

## $\rightarrow$ Home $>$ Start $>$ End

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
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\left(x, y_{1}\right) \in f \text { and }\left(x, y_{2}\right) \in f \Rightarrow y_{1}=y_{2}
\end{array}\right.
\end{array}\right.
$$

$$
f
$$

## $\rightarrow$ Home $>$ Start $>$ End

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
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\forall x \in \mathrm{X}, \exists y \in \mathrm{Y} \text { s.t. }(x, y) \in f \\
\left(x, y_{1}\right) \in f \text { and }\left(x, y_{2}\right) \in f \Rightarrow y_{1}=y_{2}
\end{array}\right.
\end{array}\right.
$$

$$
f:
$$

## $\rightarrow$ Home $>$ Start $>$ End

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
f: \mathrm{X}
$$

$$
\begin{aligned}
& f f \subset \mathrm{X} \times \mathrm{Y} \\
& \left\{\forall x \in \mathrm { X } , \exists ! y \in \mathrm { Y } \text { s.t. } ( x , y ) \in f \left\{\begin{array}{l}
\forall x \in \mathrm{X}, \exists y \in \mathrm{Y} \text { s.t. }(x, y) \in f \\
\left(x, y_{1}\right) \in f \text { and }\left(x, y_{2}\right) \in f \Rightarrow y_{1}=y_{2}
\end{array}\right.\right.
\end{aligned}
$$

## $\rightarrow$ Home $>$ Start $>$ End

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
f \subset \mathrm{X} \times \mathrm{Y} \\
\forall x \in \mathrm{X}, \exists!y \in \mathrm{Y} \text { s.t. }(x, y) \in f\left\{\begin{array}{l}
\forall x \in \mathrm{X}, \exists y \in \mathrm{Y} \text { s.t. }(x, y) \in f \\
\left(x, y_{1}\right) \in f \text { and }\left(x, y_{2}\right) \in f \Rightarrow y_{1}=y_{2}
\end{array}\right.
\end{array}\right.
$$

$$
f: \mathrm{X} \rightarrow
$$

## $\rightarrow$ Home $>$ Start $>$ End

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
f \subset \mathrm{X} \times \mathrm{Y} \\
\forall x \in \mathrm{X}, \exists!y \in \mathrm{Y} \text { s.t. }(x, y) \in f\left\{\begin{array}{l}
\forall x \in \mathrm{X}, \exists y \in \mathrm{Y} \text { s.t. }(x, y) \in f \\
\left(x, y_{1}\right) \in f \text { and }\left(x, y_{2}\right) \in f \Rightarrow y_{1}=y_{2}
\end{array}\right.
\end{array}\right.
$$

$$
f: \mathrm{X} \rightarrow \mathrm{Y},
$$

## $\rightarrow$ Home $>$ Start $>$ End

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
f \subset \mathrm{X} \times \mathrm{Y} \\
\forall x \in \mathrm{X}, \exists!y \in \mathrm{Y} \text { s.t. }(x, y) \in f\left\{\begin{array}{l}
\forall x \in \mathrm{X}, \exists y \in \mathrm{Y} \text { s.t. }(x, y) \in f \\
\left(x, y_{1}\right) \in f \text { and }\left(x, y_{2}\right) \in f \Rightarrow y_{1}=y_{2}
\end{array}\right.
\end{array}\right.
$$

$$
f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X}
$$

## $\rightarrow$ Home $>$ Start $>$ End

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
f \subset \mathrm{X} \times \mathrm{Y} \\
\forall x \in \mathrm{X}, \exists!y \in \mathrm{Y} \text { s.t. }(x, y) \in f\left\{\begin{array}{l}
\forall x \in \mathrm{X}, \exists y \in \mathrm{Y} \text { s.t. }(x, y) \in f \\
\left(x, y_{1}\right) \in f \text { and }\left(x, y_{2}\right) \in f \Rightarrow y_{1}=y_{2}
\end{array}\right. \\
f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f}
\end{array}\right.
$$

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
f \subset \mathrm{X} \times \mathrm{Y} \\
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\left(x, y_{1}\right) \in f \text { and }\left(x, y_{2}\right) \in f \Rightarrow y_{1}=y_{2}
\end{array}\right. \\
f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y}
\end{array}\right.
$$

## $\rightarrow$ Home $>$ Start $>$ End

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
f \subset \mathrm{X} \times \mathrm{Y} \\
\forall x \in \mathrm{X}, \exists!y \in \mathrm{Y} \text { s.t. }(x, y) \in f\left\{\begin{array}{l}
\forall x \in \mathrm{X}, \exists y \in \mathrm{Y} \text { s.t. }(x, y) \in f \\
\left(x, y_{1}\right) \in f \text { and }\left(x, y_{2}\right) \in f \Rightarrow y_{1}=y_{2}
\end{array}\right. \\
\quad f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y}
\end{array}\right.
$$

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
f \subset \mathrm{X} \times \mathrm{Y} \\
\forall x \in \mathrm{X}, \exists!y \in \mathrm{Y} \text { s.t. }(x, y) \in f\left\{\begin{array}{l}
\forall x \in \mathrm{X}, \exists y \in \mathrm{Y} \text { s.t. }(x, y) \in f \\
\left(x, y_{1}\right) \in f \text { and }\left(x, y_{2}\right) \in f \Rightarrow y_{1}=y_{2}
\end{array}\right. \\
\quad \begin{array}{l}
\quad \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y}
\end{array} \\
\quad \mathrm{X} \text { Domain }
\end{array}\right.
$$

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
f \subset \mathrm{X} \times \mathrm{Y} \\
\forall x \in \mathrm{X}, \exists!y \in \mathrm{Y} \text { s.t. }(x, y) \in f \\
\quad f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y} \\
\mathrm{X} \text { Domain } \\
\mathrm{Y}
\end{array}\right.
$$

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
f \subset \mathrm{X} \times \mathrm{Y} \\
\forall x \in \mathrm{X}, \quad \exists!y \in \mathrm{Y} \text { s.t. }(x, y) \in f\left\{\begin{array}{l}
\forall x \in \mathrm{X}, \exists y \in \mathrm{Y} \text { s.t. }(x, y) \in f \\
\left(x, y_{1}\right) \in f \text { and }\left(x, y_{2}\right) \in f \Rightarrow y_{1}=y_{2}
\end{array}\right. \\
\quad f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y}
\end{array}\right.
$$

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
f \subset \mathrm{X} \times \mathrm{Y} \\
\forall x \in \mathrm{X}, \exists!y \in \mathrm{Y} \text { s.t. }(x, y) \in f \\
f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y} \\
\mathrm{X} \text { Domain } \\
\mathrm{Y} \text { Codomain } \\
f
\end{array}\right.
$$

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
f \subset \mathrm{X} \times \mathrm{Y} \\
\forall x \in \mathrm{X}, \exists!y \in \mathrm{Y} \text { s.t. }(x, y) \in f\left\{\begin{array}{l}
\forall x \in \mathrm{X}, \exists y \in \mathrm{Y} \text { s.t. }(x, y) \in f \\
\left(x, y_{1}\right) \in f \text { and }\left(x, y_{2}\right) \in f \Rightarrow y_{1}=y_{2}
\end{array}\right. \\
\quad f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y} \\
\quad \mathrm{X} \text { Domain } \\
\mathrm{Y} \text { Codomain } \\
f:
\end{array}\right.
$$

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
f \subset \mathrm{X} \times \mathrm{Y} \\
\forall x \in \mathrm{X}, \exists!y \in \mathrm{Y} \text { s.t. }(x, y) \in f\left\{\begin{array}{l}
\forall x \in \mathrm{X}, \exists y \in \mathrm{Y} \text { s.t. }(x, y) \in f \\
\left(x, y_{1}\right) \in f \text { and }\left(x, y_{2}\right) \in f \Rightarrow y_{1}=y_{2}
\end{array}\right. \\
\quad f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y} \\
\quad \mathrm{X} \text { Domain } \\
\mathrm{Y} \text { Codomain } \\
\quad f: x
\end{array}\right.
$$

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
f \subset \mathrm{X} \times \mathrm{Y} \\
\forall x \in \mathrm{X}, \exists!y \in \mathrm{Y} \text { s.t. }(x, y) \in f\left\{\begin{array}{l}
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\left(x, y_{1}\right) \in f \text { and }\left(x, y_{2}\right) \in f \Rightarrow y_{1}=y_{2}
\end{array}\right. \\
\quad f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y} \\
\quad \mathrm{X} \text { Domain } \\
\mathrm{Y} \text { Codomain } \\
\quad f: x \longrightarrow
\end{array}\right.
$$

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
f \subset \mathrm{X} \times \mathrm{Y} \\
\forall x \in \mathrm{X}, \exists!y \in \mathrm{Y} \text { s.t. }(x, y) \in f\left\{\begin{array}{l}
\forall x \in \mathrm{X}, \exists y \in \mathrm{Y} \text { s.t. }(x, y) \in f \\
\left(x, y_{1}\right) \in f \text { and }\left(x, y_{2}\right) \in f \Rightarrow y_{1}=y_{2}
\end{array}\right. \\
\quad f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y} \\
\quad \mathrm{X} \text { Domain } \\
\quad \mathrm{Y} \text { Codomain } \\
\quad f: x \longrightarrow y,
\end{array}\right.
$$

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
f \subset \mathrm{X} \times \mathrm{Y} \\
\forall x \in \mathrm{X}, \exists!y \in \mathrm{Y} \text { s.t. }(x, y) \in f\left\{\begin{array}{l}
\forall x \in \mathrm{X}, \exists y \in \mathrm{Y} \text { s.t. }(x, y) \in f \\
\left(x, y_{1}\right) \in f \text { and }\left(x, y_{2}\right) \in f \Rightarrow y_{1}=y_{2}
\end{array}\right. \\
\quad f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y} \\
\mathrm{X} \text { Domain } \\
\mathrm{Y} \text { Codomain } \\
\quad f: x \longrightarrow y, x
\end{array}\right.
$$

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
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\left(x, y_{1}\right) \in f \text { and }\left(x, y_{2}\right) \in f \Rightarrow y_{1}=y_{2}
\end{array}\right. \\
\quad f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y} \\
\quad \mathrm{X} \text { Domain } \\
\mathrm{Y} \text { Codomain } \\
\quad f: x \longrightarrow y, x \xrightarrow{f}
\end{array}\right.
$$

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
f \subset \mathrm{X} \times \mathrm{Y} \\
\forall x \in \mathrm{X}, \exists!y \in \mathrm{Y} \text { s.t. }(x, y) \in f\left\{\begin{array}{l}
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\left(x, y_{1}\right) \in f \text { and }\left(x, y_{2}\right) \in f \Rightarrow y_{1}=y_{2}
\end{array}\right. \\
\quad f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y} \\
\quad \mathrm{X} \text { Domain } \\
\mathrm{Y} \text { Codomain } \\
\quad f: x \longrightarrow y, x \xrightarrow{f} y,
\end{array}\right.
$$

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
f \subset \mathrm{X} \times \mathrm{Y} \\
\forall x \in \mathrm{X}, \exists!y \in \mathrm{Y} \text { s.t. }(x, y) \in f\left\{\begin{array}{l}
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\left(x, y_{1}\right) \in f \text { and }\left(x, y_{2}\right) \in f \Rightarrow y_{1}=y_{2}
\end{array}\right. \\
\quad f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y} \\
\quad \mathrm{X} \text { Domain } \\
\mathrm{Y} \text { Codomain } \\
\quad f: x \longrightarrow y, x \xrightarrow{f} y, y
\end{array}\right.
$$

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\left\{\begin{array}{l}
f \subset \mathrm{X} \times \mathrm{Y} \\
\forall x \in \mathrm{X}, \exists!y \in \mathrm{Y} \text { s.t. }(x, y) \in f\left\{\begin{array}{l}
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\left(x, y_{1}\right) \in f \text { and }\left(x, y_{2}\right) \in f \Rightarrow y_{1}=y_{2}
\end{array}\right. \\
\quad f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y} \\
\quad \mathrm{X} \text { Domain } \\
\mathrm{Y} \text { Codomain } \\
\quad f: x \longrightarrow y, x \xrightarrow{f} y, y=
\end{array}\right.
$$

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

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\quad \mathrm{X} \text { Domain } \\
\mathrm{Y} \text { Codomain } \\
\quad f: x \longrightarrow y, x \xrightarrow{f} y, y=f
\end{array}\right.
$$

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\quad \mathrm{Y} \text { Codomain } \\
\quad f: x \longrightarrow y, x \xrightarrow{f} y, y=f(x
\end{array}\right.
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\quad \mathrm{X} \text { Domain } \\
\quad \mathrm{Y} \text { Codomain } \\
\quad f: x \longrightarrow y, x \xrightarrow{f} y, y=f(x)
\end{array}\right.
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\end{array}\right.
\end{array}\right.
$$

$$
f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y}
$$

X Domain
Y Codomain

$$
f: x \longrightarrow y, x \xrightarrow{f} y, y=f(x)
$$

$$
f(x)
$$

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

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\end{array}\right.
\end{array}\right.
$$

$$
f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y}
$$

X Domain
Y Codomain

$$
f: x \longrightarrow y, x \xrightarrow{f} y, y=f(x)
$$

$f(x)$ The value
$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
\begin{aligned}
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\end{array}\right.\right. \\
& f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y} \\
& \text { X Domain } \\
& \text { Y Codomain } \\
& f: x \longrightarrow y, x \xrightarrow{f} y, y=f(x) \\
& f(x) \text { The value of a function } f
\end{aligned}
$$

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

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\end{array}\right.
\end{array}\right.
$$

$$
f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y}
$$

X Domain
Y Codomain

$$
f: x \longrightarrow y, x \xrightarrow{f} y, y=f(x)
$$

$f(x)$ The value of a function $f$ at $x$
$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

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$$

$$
f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y}
$$

X Domain
Y Codomain

$$
f: x \longrightarrow y, x \xrightarrow{f} y, y=f(x)
$$

$f(x)$ The value of a function $f$ at $x$
$x$
$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

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\end{array}\right.
\end{array}\right.
$$

$$
f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y}
$$

X Domain
Y Codomain

$$
f: x \longrightarrow y, x \xrightarrow{f} y, y=f(x)
$$

$f(x)$ The value of a function $f$ at $x$
$x$ The independent variable
$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
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\end{array}\right.
\end{array}\right.
$$

$$
f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y}
$$

X Domain
Y Codomain

$$
f: x \longrightarrow y, x \xrightarrow{f} y, y=f(x)
$$

$f(x)$ The value of a function $f$ at $x$
$x$ The independent variable $y$
$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
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\end{array}\right.
\end{array}\right.
$$

$$
f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y}
$$

X Domain
Y Codomain

$$
f: x \longrightarrow y, x \xrightarrow{f} y, y=f(x)
$$

$f(x)$ The value of a function $f$ at $x$
$x$ The independent variable
$y$ The dependent variable
$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y}
$$

X Domain
Y Codomain

$$
f: x \longrightarrow y, x \stackrel{f}{\rightarrow} y, y=f(x)
$$

$f(x)$ The value of a function $f$ at $x$
$x$ The independent variable
$y$ The dependent variable $f(\mathrm{X})$

$$
\begin{aligned}
& \{f \subset \mathrm{X} \times \mathrm{Y} \\
& \left\{\forall x \in \mathrm { X } , \exists ! y \in \mathrm { Y } \text { s.t. } ( x , y ) \in f \left\{\begin{array}{l}
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\end{array}\right.\right.
\end{aligned}
$$

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y}
$$

X Domain
Y Codomain

$$
f: x \longrightarrow y, x \xrightarrow{f} y, y=f(x)
$$

$f(x)$ The value of a function $f$ at $x$
$x$ The independent variable
$y$ The dependent variable
$f(\mathrm{X})=$

$$
\begin{aligned}
& \{f \subset \mathrm{X} \times \mathrm{Y} \\
& \left\{\forall x \in \mathrm { X } , \exists ! y \in \mathrm { Y } \text { s.t. } ( x , y ) \in f \left\{\begin{array}{l}
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\end{array}\right.\right.
\end{aligned}
$$

$\mathrm{X} \times \mathrm{Y}=\{(x, y) \mid x \in \mathrm{X}, y \in \mathrm{Y}\}:$ The Cartesian product $\mathrm{X} \times \mathrm{Y}$ $f$ is a function from X to Y .

$$
f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y}
$$

X Domain
Y Codomain

$$
f: x \longrightarrow y, x \xrightarrow{f} y, y=f(x)
$$

$f(x)$ The value of a function $f$ at $x$
$x$ The independent variable
$y$ The dependent variable
$f(\mathrm{X})=\{$

$$
\begin{aligned}
& \{f \subset \mathrm{X} \times \mathrm{Y} \\
& \left\{\forall x \in \mathrm { X } , \exists ! y \in \mathrm { Y } \text { s.t. } ( x , y ) \in f \left\{\begin{array}{l}
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\end{aligned}
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$$
f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y}
$$

X Domain
Y Codomain

$$
f: x \longrightarrow y, x \xrightarrow{f} y, y=f(x)
$$

$f(x)$ The value of a function $f$ at $x$
$x$ The independent variable
$y$ The dependent variable
$f(\mathrm{X})=\{y$

$$
\begin{aligned}
& \{f \subset \mathrm{X} \times \mathrm{Y} \\
& \left\{\forall x \in \mathrm { X } , \exists ! y \in \mathrm { Y } \text { s.t. } ( x , y ) \in f \left\{\begin{array}{l}
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$$
f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y}
$$

X Domain
Y Codomain

$$
f: x \longrightarrow y, x \xrightarrow{f} y, y=f(x)
$$

$f(x)$ The value of a function $f$ at $x$
$x$ The independent variable
$y$ The dependent variable

$$
f(X)=\{y \mid
$$

$$
\begin{aligned}
& \{f \subset \mathrm{X} \times \mathrm{Y} \\
& \left\{\forall x \in \mathrm { X } , \exists ! y \in \mathrm { Y } \text { s.t. } ( x , y ) \in f \left\{\begin{array}{l}
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$$
f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y}
$$

X Domain
Y Codomain

$$
f: x \longrightarrow y, x \xrightarrow{f} y, y=f(x)
$$

$f(x)$ The value of a function $f$ at $x$
$x$ The independent variable
$y$ The dependent variable

$$
f(\mathrm{X})=\{y \mid \exists x
$$

$$
\begin{aligned}
& \{f \subset \mathrm{X} \times \mathrm{Y} \\
& \left\{\forall x \in \mathrm { X } , \exists ! y \in \mathrm { Y } \text { s.t. } ( x , y ) \in f \left\{\begin{array}{l}
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f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y}
$$

X Domain
Y Codomain

$$
f: x \longrightarrow y, x \xrightarrow{f} y, y=f(x)
$$

$f(x)$ The value of a function $f$ at $x$
$x$ The independent variable
$y$ The dependent variable
$f(\mathrm{X})=\{y \mid \exists x \in$

$$
\begin{aligned}
& \{f \subset \mathrm{X} \times \mathrm{Y} \\
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f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y}
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X Domain
Y Codomain

$$
f: x \longrightarrow y, x \xrightarrow{f} y, y=f(x)
$$

$f(x)$ The value of a function $f$ at $x$
$x$ The independent variable
$y$ The dependent variable
$f(\mathrm{X})=\{y \mid \exists x \in \mathrm{X}$

$$
\begin{aligned}
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X Domain
Y Codomain

$$
f: x \longrightarrow y, x \xrightarrow{f} y, y=f(x)
$$

$f(x)$ The value of a function $f$ at $x$
$x$ The independent variable
$y$ The dependent variable
$f(\mathrm{X})=\{y \mid \exists x \in \mathrm{X}$ s.t.

$$
\begin{aligned}
& \{f \subset \mathrm{X} \times \mathrm{Y} \\
& \left\{\forall x \in \mathrm { X } , \exists ! y \in \mathrm { Y } \text { s.t. } ( x , y ) \in f \left\{\begin{array}{l}
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X Domain
Y Codomain

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f: x \longrightarrow y, x \xrightarrow{f} y, y=f(x)
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$f(x)$ The value of a function $f$ at $x$
$x$ The independent variable
$y$ The dependent variable

$$
f(\mathrm{X})=\{y \mid \exists x \in \mathrm{X} \text { s.t. } y=f(x)
$$

$$
\begin{aligned}
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$$
f: \mathrm{X} \rightarrow \mathrm{Y}, \mathrm{X} \xrightarrow{f} \mathrm{Y}
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X Domain
Y Codomain

$$
f: x \longrightarrow y, x \xrightarrow{f} y, y=f(x)
$$

$f(x)$ The value of a function $f$ at $x$
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Image of $f$ or Range

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Image of $f$ or Range of $f$

Github:
https://min7014.github.io/math20190810112.html

## Click or paste URL into the URL search bar, and you can see a picture moving.

