$$\int \csc x \; dx$$

$\csc x$

$$\csc x \quad =$$

$$\csc x = \frac{1}{\sin x}$$

$$\csc x = \frac{1}{\sin x} = \frac{\sin x}{\sin^2 x}$$

$$\csc x = \frac{1}{\sin x} = \frac{\sin x}{\sin^2 x} = \frac{\sin x}{1 - \cos^2 x}$$

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$\int \csc x \; dx$

$$\begin{aligned}\csc x &= \frac{1}{\sin x} = \frac{\sin x}{\sin^2 x} = \frac{\sin x}{1-\cos^2 x} = \frac{\sin x}{(1-\cos x)(1+\cos x)} \\ &= \frac{1}{2}\left(\frac{\sin x}{1-\cos x} + \frac{\sin x}{1+\cos x}\right)\end{aligned}$$

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$$\int \csc x \, dx$$

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&= -\frac{1}{2}\ln\left\{\frac{(1+\cos x)^2}{\sin^2 x}\right\} + c = -\frac{1}{2}\ln\left(\frac{1+\cos x}{\sin x}\right)^2 + c
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END