Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

▸ Start

Start

$x_1 > 0, \ldots, x_n > 0$

Start

$x_1 > 0, \ldots, x_n > 0$

$$\frac{x_1 + \cdots + x_n}{n}$$

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

Start

$x_1 > 0, \ldots, x_n > 0$

$$\frac{x_1 + \cdots + x_n}{n} \geq$$

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Start

$x_1 > 0, \ldots, x_n > 0$

$$\frac{x_1 + \cdots + x_n}{n} \geq \sqrt[n]{x_1 \times \cdots \times x_n}$$

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$x_1 > 0, \ldots, x_n > 0$

$$\frac{x_1 + \cdots + x_n}{n} \geq \sqrt[n]{x_1 \times \cdots \times x_n} \geq$$

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$x_1 > 0, \ldots, x_n > 0$

$$\frac{x_1 + \cdots + x_n}{n} \geq \sqrt[n]{x_1 \times \cdots \times x_n} \geq \frac{n}{\dfrac{1}{x_1} + \cdots + \dfrac{1}{x_n}}$$

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$x_1 > 0, \ldots, x_n > 0$

$$\frac{x_1 + \cdots + x_n}{n} \geq \sqrt[n]{x_1 \times \cdots \times x_n} \geq \frac{n}{\dfrac{1}{x_1} + \cdots + \dfrac{1}{x_n}}$$

Proof of first inequality

Proof of second inequality

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$x_1 > 0, \ldots, x_n > 0$

$$\frac{x_1 + \cdots + x_n}{n} \geq \sqrt[n]{x_1 \times \cdots \times x_n} \geq \frac{n}{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}$$

▸ Proof of first inequality

▸ Proof of second inequality

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- Proof of first inequality

Start

- Proof of first inequality
  If $n = 2$.

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- Proof of first inequality

  If $n = 2$.

  $\left(\frac{x_1 + x_2}{2}\right)^2 - (\sqrt{x_1 x_2})^2$

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Start

- Proof of first inequality

  If $n = 2$.

  $$\left(\frac{x_1 + x_2}{2}\right)^2 - (\sqrt{x_1 x_2})^2 \quad = \quad \frac{x_1^2 + 2x_1x_2 + x_2^2}{4} - x_1x_2$$

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- Proof of first inequality

  If $n = 2$.

$$\begin{aligned}\left(\frac{x_1 + x_2}{2}\right)^2 - (\sqrt{x_1 x_2})^2 &= \frac{x_1^2 + 2x_1x_2 + x_2^2}{4} - x_1x_2 \\ &= \frac{x_1^2 - 2x_1x_2 + x_2^2}{4}\end{aligned}$$

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

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- Proof of first inequality

  If $n = 2$.

$$
\begin{aligned}
\left(\frac{x_1 + x_2}{2}\right)^2 - \left(\sqrt{x_1 x_2}\right)^2 &= \frac{x_1^2 + 2x_1x_2 + x_2^2}{4} - x_1x_2 \\
&= \frac{x_1^2 - 2x_1x_2 + x_2^2}{4} \\
&= \frac{(x_1 - x_2)^2}{4}
\end{aligned}
$$

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

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- Proof of first inequality

  If $n = 2$.

$$\begin{aligned}\left(\frac{x_1+x_2}{2}\right)^2 - (\sqrt{x_1x_2})^2 &= \frac{x_1^2+2x_1x_2+x_2^2}{4} - x_1x_2 \\ &= \frac{x_1^2-2x_1x_2+x_2^2}{4} \\ &= \frac{(x_1-x_2)^2}{4} \geq\end{aligned}$$

Start

- Proof of first inequality

  If $n = 2$.

$$\begin{aligned}\left(\frac{x_1+x_2}{2}\right)^2-(\sqrt{x_1x_2})^2 &= \frac{x_1^2+2x_1x_2+x_2^2}{4}-x_1x_2\\ &= \frac{x_1^2-2x_1x_2+x_2^2}{4}\\ &= \frac{(x_1-x_2)^2}{4}\geq 0\end{aligned}$$

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

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- Proof of first inequality

If $n = 2$.

$$\left(\frac{x_1 + x_2}{2}\right)^2 - (\sqrt{x_1 x_2})^2 = \frac{x_1^2 + 2x_1x_2 + x_2^2}{4} - x_1x_2$$

$$= \frac{x_1^2 - 2x_1x_2 + x_2^2}{4}$$

$$= \frac{(x_1 - x_2)^2}{4} \geq 0$$

$$\therefore \frac{x_1 + x_2}{2}$$

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- Proof of first inequality

If $n = 2$.

$$\begin{aligned}\left(\frac{x_1+x_2}{2}\right)^2-(\sqrt{x_1x_2})^2 &= \frac{x_1^2+2x_1x_2+x_2^2}{4}-x_1x_2\\ &= \frac{x_1^2-2x_1x_2+x_2^2}{4}\\ &= \frac{(x_1-x_2)^2}{4}\geq 0\end{aligned}$$

$$\therefore \frac{x_1+x_2}{2}\geq$$

Start

- Proof of first inequality

If $n = 2$.

$$\begin{aligned}\left(\frac{x_1+x_2}{2}\right)^2-(\sqrt{x_1x_2})^2 &= \frac{x_1^2+2x_1x_2+x_2^2}{4}-x_1x_2\\ &= \frac{x_1^2-2x_1x_2+x_2^2}{4}\\ &= \frac{(x_1-x_2)^2}{4}\geq 0\end{aligned}$$

$$\therefore \frac{x_1+x_2}{2}\geq\sqrt{x_1x_2}$$

Start

- Proof of first inequality

If $n = 2$.

$$\left(\frac{x_1 + x_2}{2}\right)^2 - (\sqrt{x_1 x_2})^2 = \frac{x_1^2 + 2x_1x_2 + x_2^2}{4} - x_1 x_2$$

$$= \frac{x_1^2 - 2x_1x_2 + x_2^2}{4}$$

$$= \frac{(x_1 - x_2)^2}{4} \geq 0$$

$$\therefore \frac{x_1 + x_2}{2} \geq \sqrt{x_1 x_2}$$

$$n = 2$$

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

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- Proof of first inequality

  If $n = 2$.

$$\begin{aligned}\left(\frac{x_1 + x_2}{2}\right)^2 - (\sqrt{x_1 x_2})^2 &= \frac{x_1^2 + 2x_1x_2 + x_2^2}{4} - x_1x_2 \\ &= \frac{x_1^2 - 2x_1x_2 + x_2^2}{4} \\ &= \frac{(x_1 - x_2)^2}{4} \geq 0\end{aligned}$$

$$\therefore \frac{x_1 + x_2}{2} \geq \sqrt{x_1 x_2}$$

$n = 2$ is true.

Start

- Proof of first inequality

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

Start

- Proof of first inequality
  Assume

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

Start

- Proof of first inequality
  Assume $n = 2^{k-1}$

Start

- Proof of first inequality
  Assume $n = 2^{k-1}$ is true.

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

Start

- Proof of first inequality
  Assume $n = 2^{k-1}$ is true.

$$\frac{x_1 + \cdots + x_{2^k}}{2^k}$$

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- Proof of first inequality
  Assume $n = 2^{k-1}$ is true.

$$\frac{x_1 + \cdots + x_{2^k}}{2^k} =$$

Start

- Proof of first inequality
  Assume $n = 2^{k-1}$ is true.

$$\frac{x_1 + \cdots + x_{2^k}}{2^k} = \frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}}$$

Start

- Proof of first inequality
  Assume $n = 2^{k-1}$ is true.

$$\frac{x_1 + \cdots + x_{2^k}}{2^k} = \frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} +$$

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

Start

- Proof of first inequality
  Assume $n = 2^{k-1}$ is true.

$$\frac{x_1 + \cdots + x_{2^k}}{2^k} = \frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}$$

Start

- Proof of first inequality
  Assume $n = 2^{k-1}$ is true.

$$\frac{x_1 + \cdots + x_{2^k}}{2^k} = \frac{\frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2}$$

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

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$$\frac{x_1 + \cdots + x_{2^k}}{2^k} = \frac{\frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2}$$

$$\geq$$

Start

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  Assume $n = 2^{k-1}$ is true.

$$\frac{x_1 + \cdots + x_{2^k}}{2^k} = \frac{\frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2}$$

$$\geq \sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}}$$

Start

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$$\geq \sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} +$$

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$$\frac{x_1 + \cdots + x_{2^k}}{2^k} = \frac{\frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2}$$

$$\geq \sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}$$

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  Assume $n = 2^{k-1}$ is true.

$$\frac{x_1 + \cdots + x_{2^k}}{2^k} = \frac{\frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2}$$

$$\geq \frac{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}{2}$$

Start

- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\frac{x_1 + \cdots + x_{2^k}}{2^k} = \frac{\frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2}$$

$$\geq \frac{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}{2}$$

$$\geq$$

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$$\frac{x_1 + \cdots + x_{2^k}}{2^k} = \frac{\frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2}$$

$$\geq \frac{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}{2}$$

$$\geq \sqrt{}$$

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

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- Proof of first inequality

  Assume $n = 2^{k-1}$ is true.

$$\frac{x_1 + \cdots + x_{2^k}}{2^k} = \frac{\frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2}$$

$$\geq \frac{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}{2}$$

$$\geq \sqrt{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}}}$$

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

Start

- Proof of first inequality

  Assume $n = 2^{k-1}$ is true.

$$
\begin{aligned}
\frac{x_1 + \cdots + x_{2^k}}{2^k} &= \frac{\frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2} \\
&\geq \frac{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}{2} \\
&\geq \sqrt{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}}\ \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}
\end{aligned}
$$

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

Start

- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\begin{aligned}
\frac{x_1 + \cdots + x_{2^k}}{2^k} &= \frac{\dfrac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \dfrac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2} \\
&\geq \frac{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}{2} \\
&\geq \sqrt{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}}\ \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}} \\
&=
\end{aligned}$$

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▸ Start

- Proof of first inequality

  Assume $n = 2^{k-1}$ is true.

$$\begin{aligned}
\frac{x_1 + \cdots + x_{2^k}}{2^k} &= \frac{\dfrac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \dfrac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2} \\
&\geq \frac{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}{2} \\
&\geq \sqrt{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}}\ \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}} \\
&= \sqrt[2^k]{x_1 \cdots x_{2^k}}
\end{aligned}$$

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

Start

- Proof of first inequality
  Assume $n = 2^{k-1}$ is true.

$$
\begin{aligned}
\frac{x_1 + \cdots + x_{2^k}}{2^k} &= \frac{\frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2} \\
&\geq \frac{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}{2} \\
&\geq \sqrt{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}}\ \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}} \\
&= \sqrt[2^k]{x_1 \cdots x_{2^k}}
\end{aligned}
$$

$$\therefore \frac{x_1 + \cdots + x_{2^k}}{2^k}$$

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▸ Start

- Proof of first inequality
  Assume $n = 2^{k-1}$ is true.

$$\frac{x_1 + \cdots + x_{2^k}}{2^k} = \frac{\frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2}$$

$$\geq \frac{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}{2}$$

$$\geq \sqrt{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}$$

$$= \sqrt[2^k]{x_1 \cdots x_{2^k}}$$

$$\therefore \frac{x_1 + \cdots + x_{2^k}}{2^k} \geq$$

Start

- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$\begin{aligned}
\frac{x_1 + \cdots + x_{2^k}}{2^k} &= \frac{\frac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \frac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2} \\
&\geq \frac{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}{2} \\
&\geq \sqrt{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}}\ \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}} \\
&= \sqrt[2^k]{x_1 \cdots x_{2^k}}
\end{aligned}$$

$$\therefore \frac{x_1 + \cdots + x_{2^k}}{2^k} \geq \sqrt[2^k]{x_1 \cdots x_{2^k}}$$

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

Start

- Proof of first inequality

Assume $n = 2^{k-1}$ is true.

$$
\begin{aligned}
\frac{x_1 + \cdots + x_{2^k}}{2^k} &= \frac{\dfrac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \dfrac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2} \\
&\geq \frac{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}{2} \\
&\geq \sqrt{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}}\ \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}} \\
&= \sqrt[2^k]{x_1 \cdots x_{2^k}}
\end{aligned}
$$

$$\therefore \frac{x_1 + \cdots + x_{2^k}}{2^k} \geq \sqrt[2^k]{x_1 \cdots x_{2^k}}$$

$$n = 2^k$$

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

▸ Start

- Proof of first inequality
  Assume $n = 2^{k-1}$ is true.

$$\begin{aligned}
\frac{x_1 + \cdots + x_{2^k}}{2^k} &= \frac{\dfrac{x_1 + \cdots + x_{2^{k-1}}}{2^{k-1}} + \dfrac{x_{2^{k-1}+1} + \cdots + x_{2^k}}{2^{k-1}}}{2} \\
&\geq \frac{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}} + \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}}{2} \\
&\geq \sqrt{\sqrt[2^{k-1}]{x_1 \cdots x_{2^{k-1}}}\ \sqrt[2^{k-1}]{x_{2^{k-1}+1} \cdots x_{2^k}}} \\
&= \sqrt[2^k]{x_1 \cdots x_{2^k}}
\end{aligned}$$

$$\therefore \frac{x_1 + \cdots + x_{2^k}}{2^k} \geq \sqrt[2^k]{x_1 \cdots x_{2^k}}$$

$$n = 2^k \text{ is true.}$$

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Start

- Proof of first inequality

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

Start

- Proof of first inequality
  Let $m = 2^l$

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Start

- Proof of first inequality
  Let $m = 2^l$ such that

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Start

- Proof of first inequality
  Let $m = 2^l$ such that $n < m$.

Start

- Proof of first inequality
  Let $m = 2^l$ such that $n < m$.

  α

Start

- Proof of first inequality
  Let $m = 2^l$ such that $n < m$.

$$\alpha \quad =$$

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

Start

- Proof of first inequality
  Let $m = 2^l$ such that $n < m$.

$$\alpha = \frac{x_1 + \cdots + x_n}{n}$$

▸ Start

- Proof of first inequality

  Let $m = 2^l$ such that $n < m$.

$$\alpha = \frac{x_1 + \cdots + x_n}{n} =$$

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

Start

- Proof of first inequality
  Let $m = 2^l$ such that $n < m$.

$$\alpha = \frac{x_1 + \cdots + x_n}{n} = \frac{m}{n}$$

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Start

- Proof of first inequality

  Let $m = 2^l$ such that $n < m$.

$$\alpha = \frac{x_1 + \cdots + x_n}{n} = \frac{m}{n}(x_1 + \cdots + x_n)$$

Start

- Proof of first inequality

  Let $m = 2^l$ such that $n < m$.

$$\alpha = \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m}$$

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- Proof of first inequality
  Let $m = 2^l$ such that $n < m$.

$$\alpha = \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m}$$

$$=$$

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

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- Proof of first inequality

  Let $m = 2^l$ such that $n < m$.

$$\alpha = \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m}$$

$$= \quad x_1 + \cdots + x_n$$

Start

- Proof of first inequality

  Let $m = 2^l$ such that $n < m$.

$$\alpha = \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m}$$

$$= \quad x_1 + \cdots + x_n +$$

Start

- Proof of first inequality
  Let $m = 2^l$ such that $n < m$.

$$\alpha = \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m}$$

$$= \frac{x_1 + \cdots + x_n + \frac{m-n}{n}}{}$$

Start

- Proof of first inequality

  Let $m = 2^l$ such that $n < m$.

$$\alpha = \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m}$$

$$= \frac{x_1 + \cdots + x_n + \frac{m-n}{n}(x_1 + \cdots + x_n)}{}$$

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$$\alpha = \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m}$$
$$= \frac{x_1 + \cdots + x_n + \frac{m-n}{n}(x_1 + \cdots + x_n)}{m}$$

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

Start

- Proof of first inequality
  Let $m = 2^l$ such that $n < m$.

$$\begin{aligned}\alpha &= \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m} \\ &= \frac{x_1 + \cdots + x_n + \frac{m-n}{n}(x_1 + \cdots + x_n)}{m} \\ &= \end{aligned}$$

▸ Start

- Proof of first inequality
  Let $m = 2^l$ such that $n < m$.

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Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

Start

- Proof of first inequality

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&= \frac{x_1 + \cdots + x_n + (m-n)\alpha}{m}
\end{aligned}
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Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

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Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

▸ Start

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Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

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&= \frac{x_1 + \cdots + x_n + (m-n)\alpha}{m} \\
&= \frac{x_1 + \cdots + x_n + \overbrace{\alpha + \cdots + \alpha}^{m-n}}{m} \geq
\end{aligned}
$$

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

▸ Start

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&= \frac{x_1 + \cdots + x_n + (m-n)\alpha}{m} \\
&= \frac{x_1 + \cdots + x_n + \overbrace{\alpha + \cdots + \alpha}^{m-n}}{m} \geq \sqrt[m]{x_1 \cdots x_n \alpha^{m-n}}
\end{aligned}$$

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\end{aligned}$$

$$\alpha^m \geq x_1 \cdots x_n \alpha^{m-n} \quad ,$$

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

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&= \frac{x_1 + \cdots + x_n + (m-n)\alpha}{m} \\
&= \frac{x_1 + \cdots + x_n + \overbrace{\alpha + \cdots + \alpha}^{m-n}}{m} \geq \sqrt[m]{x_1 \cdots x_n \alpha^{m-n}}
\end{aligned}$$

$$\alpha^m \geq x_1 \cdots x_n \alpha^{m-n} \ , \ \ \alpha^n \geq x_1 \cdots x_n \ ,$$

▸ Start

- Proof of first inequality

Let $m = 2^l$ such that $n < m$.

$$\begin{aligned}
\alpha &= \frac{x_1 + \cdots + x_n}{n} = \frac{\frac{m}{n}(x_1 + \cdots + x_n)}{m} \\
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&= \frac{x_1 + \cdots + x_n + (m-n)\alpha}{m} \\
&= \frac{x_1 + \cdots + x_n + \overbrace{\alpha + \cdots + \alpha}^{m-n}}{m} \geq \sqrt[m]{x_1 \cdots x_n \alpha^{m-n}}
\end{aligned}$$

$$\alpha^m \geq x_1 \cdots x_n \alpha^{m-n} \;,\; \alpha^n \geq x_1 \cdots x_n \;,\; \alpha \geq \sqrt[n]{x_1 \cdots x_n}$$

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

Start

- Proof of first inequality

  Let $m = 2^l$ such that $n < m$.

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&= \frac{x_1 + \cdots + x_n + \overbrace{\alpha + \cdots + \alpha}^{m-n}}{m} \geq \sqrt[m]{x_1 \cdots x_n \alpha^{m-n}}
\end{aligned}$$

$$\alpha^m \geq x_1 \cdots x_n \alpha^{m-n} \quad , \quad \alpha^n \geq x_1 \cdots x_n \quad , \quad \alpha \geq \sqrt[n]{x_1 \cdots x_n}$$

$$\therefore \frac{x_1 + \cdots + x_n}{n} \geq \sqrt[n]{x_1 \cdots x_n} \quad (n \text{ is true.})$$

Start

- Proof of second inequality

Start

- Proof of second inequality

$$\frac{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}{n}$$

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

Start

- Proof of second inequality

$$\frac{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}{n} \geq$$

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$$\frac{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}{n} \geq \sqrt[n]{\frac{1}{x_1} \cdots \frac{1}{x_n}}$$

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

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$$\frac{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}{n} \geq \sqrt[n]{\frac{1}{x_1} \cdots \frac{1}{x_n}}$$

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Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

Start

- Proof of second inequality

$$\frac{\frac{1}{x_1}+\cdots+\frac{1}{x_n}}{n} \geq \sqrt[n]{\frac{1}{x_1}\cdots\frac{1}{x_n}}$$

$$\frac{\frac{1}{x_1}+\cdots+\frac{1}{x_n}}{n} \geq$$

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

▸ Start

- Proof of second inequality

$$\frac{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}{n} \geq \sqrt[n]{\frac{1}{x_1} \cdots \frac{1}{x_n}}$$

$$\frac{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}{n} \geq \frac{1}{\sqrt[n]{x_1 \cdots x_n}}$$

Inequality of Arithmetic and Geometric and Harmonic Means for n non-negative real numbers

Start

- Proof of second inequality

$$\frac{\frac{1}{x_1}+\cdots+\frac{1}{x_n}}{n} \geq \sqrt[n]{\frac{1}{x_1}\cdots\frac{1}{x_n}}$$

$$\frac{\frac{1}{x_1}+\cdots+\frac{1}{x_n}}{n} \geq \frac{1}{\sqrt[n]{x_1\cdots x_n}}$$

$$\therefore \sqrt[n]{x_1 \times \cdots \times x_n} \geq$$

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$$\therefore \sqrt[n]{x_1\times\cdots\times x_n} \geq \frac{n}{\frac{1}{x_1}+\cdots+\frac{1}{x_n}}$$

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