연쇄법칙
(The Chain Rule)

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Theorem

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$$F'(x)$$

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$$F'(x) = \lim_{h \to 0} \frac{f(g(x+h)) - f(g(x))}{h}$$

▸ Start

Proof.

$$\varepsilon_1(h) = \begin{cases} \dfrac{g(x+h)-g(x)}{h} - g'(x) & , \quad h \neq 0 \\ 0 & , \quad h = 0 \end{cases}$$

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