압착정리
(The Squeeze Theorem)

Start

Theorem

Start

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$f(x) \leq g(x) \leq h(x)(0 < |x-a| < \delta_0)$, $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$

$$\lim_{x \to a} g(x) = L$$

Proof.

$\epsilon > 0$

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Start

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Start

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$\therefore \forall \epsilon > 0$, $\exists \delta > 0$

Start

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Home