곱의 극한은 극한의 곱이다.
(The limit of a product is the product of the limits.)

The limit of a product is the product of the limits.

▸ Start

Theorem

The limit of a product is the product of the limits.

Start

Theorem

$$\lim_{x \to a} f(x) = L$$

The limit of a product is the product of the limits.

Start

Theorem

$\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

▸ Start

Theorem

$\lim_{x \to a} f(x) = L,\ \lim_{x \to a} g(x) = M$

$$\lim_{x \to a} \{f(x)g(x)\}$$

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▸ Start

Theorem

$\lim_{x\to a} f(x) = L$, $\lim_{x\to a} g(x) = M$

$$\lim_{x\to a}\{f(x)g(x)\} = LM$$

The limit of a product is the product of the limits.

Start

Theorem

$\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

$$\lim_{x \to a} \{f(x)g(x)\} = LM$$

Proof.

The limit of a product is the product of the limits.

Start

Theorem

$\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

$$\lim_{x \to a} \{f(x)g(x)\} = LM$$

Proof.

$\epsilon > 0$

The limit of a product is the product of the limits.

▸ Start

Theorem

$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$

$$\lim_{x \to a} \{f(x)g(x)\} = LM$$

Proof.

$\epsilon > 0$

$|f(x)g(x) - LM|$

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Start

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$\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

$$\lim_{x \to a} \{f(x)g(x)\} = LM$$

Proof.

$\epsilon > 0$

$$|f(x)g(x) - LM| = |f(x)g(x) - f(x)M + f(x)M - LM|$$

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$\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

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Proof.

$\epsilon > 0$

$$\begin{aligned} |f(x)g(x) - LM| &= |f(x)g(x) - f(x)M + f(x)M - LM| \\ &\leq |f(x)g(x) - f(x)M| + |f(x)M - LM| \end{aligned}$$

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$$\begin{aligned} |f(x)g(x) - LM| &= |f(x)g(x) - f(x)M + f(x)M - LM| \\ &\le |f(x)g(x) - f(x)M| + |f(x)M - LM| \ (\because \text{ The Triangle Inequality}) \\ &= |f(x)| \cdot |g(x) - M| + |Mf(x) - ML| \end{aligned}$$

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$|f(x)|$

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$|f(x)| = |f(x) - L + L|$

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$|f(x)| = |f(x) - L + L| \leq |f(x) - L| + |L|$

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$|f(x)| = |f(x) - L + L| \leq |f(x) - L| + |L| < 1 + |L|$

$\exists \delta_2 > 0$ s.t. $0 < |x - a| < \delta_2 \Rightarrow |g(x) - M| < \dfrac{\epsilon}{2(1 + |L|)}$

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