합의 극한은 극한의 합이다.
(The limit of a sum is the sum of the limits.)

The limit of a sum is the sum of the limits.

Start

Theorem

The limit of a sum is the sum of the limits.

Start

Theorem

$\lim_{x \to a} f(x) = L$

The limit of a sum is the sum of the limits.

Start

Theorem

$\lim_{x \to a} f(x) = L, \lim_{x \to a} g(x) = M$

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Start

Theorem

$\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

$$\lim_{x \to a} \{f(x) + g(x)\}$$

The limit of a sum is the sum of the limits.

Start

Theorem

$\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

$$\lim_{x \to a} \{f(x) + g(x)\} = L + M$$

The limit of a sum is the sum of the limits.

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Theorem

$\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

$$\lim_{x \to a} \{f(x) + g(x)\} = L + M$$

Proof.

The limit of a sum is the sum of the limits.

Start

Theorem

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Proof.

$\epsilon > 0$

The limit of a sum is the sum of the limits.

▸ Start

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Proof.

$\epsilon > 0$

$|\{f(x) + g(x)\} - (L + M)|$

Start

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$$\lim_{x \to a} \{f(x) + g(x)\} = L + M$$

Proof.

$\epsilon > 0$

$$|\{f(x) + g(x)\} - (L + M)| = |\{f(x) - L\} + \{g(x) - M\}|$$

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$$\begin{aligned} |\{f(x) + g(x)\} - (L + M)| &= |\{f(x) - L\} + \{g(x) - M\}| \\ &\leq |f(x) - L| + |g(x) - M| \end{aligned}$$

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$\exists \delta_1 > 0$ s.t. $0 < |x - a| < \delta_1$

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$$\begin{aligned} |\{f(x)+g(x)\} - (L+M)| &= |\{f(x)-L\} + \{g(x)-M\}| \\ &\leq |f(x)-L| + |g(x)-M| \ (\because \text{The Triangle Inequality}) \end{aligned}$$

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$$\begin{aligned} |\{f(x)+g(x)\}-(L+M)| &= |\{f(x)-L\}+\{g(x)-M\}| \\ &\leq |f(x)-L|+|g(x)-M| \ (\because \text{The Triangle Inequality}) \end{aligned}$$

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▸ Start

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Start

Theorem

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δ

The limit of a sum is the sum of the limits.

Start

Theorem

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$\delta = \min(\delta_1, \delta_2)$

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Start

Theorem

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$0 < |x - a| < \delta \Rightarrow |\{f(x) + g(x)\} - (L + M)| < \epsilon$

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▸ Start

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$\therefore$

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$0 < |x - a| < \delta \Rightarrow |\{f(x) + g(x)\} - (L + M)| < \epsilon$

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$\epsilon > 0$

$$\begin{aligned} |\{f(x) + g(x)\} - (L + M)| &= |\{f(x) - L\} + \{g(x) - M\}| \\ &\leq |f(x) - L| + |g(x) - M| \ (\because \text{The Triangle Inequality}) \end{aligned}$$

$\exists \delta_1 > 0$ s.t. $0 < |x - a| < \delta_1 \Rightarrow |f(x) - L| < \frac{\epsilon}{2}$ $(\because \lim_{x \to a} f(x) = L)$

$\exists \delta_2 > 0$ s.t. $0 < |x - a| < \delta_2 \Rightarrow |g(x) - M| < \frac{\epsilon}{2}$ $(\because \lim_{x \to a} g(x) = M)$

$\delta = \min(\delta_1, \delta_2)$

$0 < |x - a| < \delta \Rightarrow |\{f(x) + g(x)\} - (L + M)| < \epsilon$

$\therefore \forall \epsilon > 0, \exists \delta > 0$ s.t. $0 < |x - a| < \delta \Rightarrow |\{f(x) + g(x)\} - (L + M)| < \epsilon$

$\square$

The limit of a sum is the sum of the limits.

Home